Frounhofer Diffraction due to Double Slit

Bp, Qsnpgw_ P_lg
E scqr D_asjrw
Bcn_rr k c l r md N f wqgaq
K_e_b f K_f gj_ Amjjcec* NS
C k _gj gb+ q s npgw_,n f wqgaq > e k _gj,am k

FRAUNHOFER DIFFRACTION DUE TO DOUBLE SLIT

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon two parallel slits AB and CD, each of width *b* and their separation as *d*. The distance between the corresponding points of two slits will be *(b+d)*.

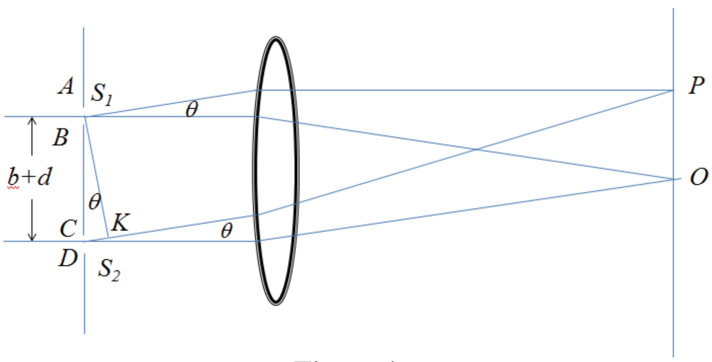

Figure 1

Suppose each slit diffracts the beam in a direction making an angle θ with the direction of incident beam. From the theory of diffraction at single slit, the resultant amplitude will be

$$\frac{A\ Sin\ \alpha}{\alpha}$$

Where $$\alpha = \frac{\pi b Sin\ \theta}{\lambda}$$

Now consider the two slits equivalent to two coherent sources, placed at the middle points S_1 and $S_2$of the slits and each sending a wavelet of amplitude $\frac{A\ Sin\ \alpha}{\alpha}$.

Therefore, the resultant amplitude at point P on the screen will be the result of the interference between two waves of same amplitude $\frac{A\ Sin\ \alpha}{\alpha}$ and having a phase difference δ.

∴ Path difference between the wavelets coming from S_1 and S_2in direction θ is given by

$$S_2K = (b+d)\ sin\theta$$

Phase difference = $\frac{2\pi}{\lambda}$ x path difference = $\frac{2\pi}{\lambda}(b+d)\sin\theta) = 2\beta$

Resultant amplitude R at point P can be obtained by vector addition method as

$$I = R^2 = 4A^2 \frac{Sin^2\alpha\ Cos^2\beta}{\alpha^2} \qquad .... (1)$$

Here $\frac{Sin^2\alpha}{\alpha^2}$ gives the diffraction pattern due to each individual slit and $Cos^2\beta$ gives the interference pattern due to double slit. $\frac{Sin^2\alpha}{\alpha^2}$ gives a central maximum in the direction β= 0, having alternate minima and secondary maxima of decreasing intensity on either side.

The minima are obtained in the directions given by

$$Sin\alpha = 0 \qquad \text{or} \qquad \alpha = \pm m\pi$$

∵ $$\alpha = \frac{\pi b\ Sin\ \theta}{\lambda}$$

∴ $$b\,Sin\theta = \pm m\pi \qquad .... (2)$$

Where m = 1, 2, 3…. (except zero).

The term $Cos^2\beta$ in the intensity pattern gives a set of equidistant dark and bright fringes.

$$Cos^2\beta = 1$$

∴ $$\beta = \pm\ n\pi$$

$$\frac{\pi}{\lambda}(b+d)\sin\theta = \pm\ n\pi$$

$$(b+d)\sin\theta = \pm\ n\lambda \qquad .... (3)$$

Where n = 0, 1, 2, 3…., correspond to zero-, first-, second- etc. order Maxima.

Missing Order

In the output intensity pattern of a double slit, for certain values of d, few interference maxima become absent.

As, the directions of interference maxima are given by

$$(b + d) \sin \theta = n\lambda \qquad \text{....... (4)}$$

The directions of diffraction minima are given by

$$b \sin \theta = m\lambda \qquad \text{....... (5)}$$

If the values of *b* and *d* are such that both the equations are satisfied for the same value of a, then a certain interference maximum will overlap the diffraction minimum and hence the spectrum order will be missing (absent).

Dividing equation (4) by equation (5), we get,

$$\frac{b + d}{b} = \frac{n}{m} \qquad \text{....... (6)}$$

If $b=d$

$$\frac{n}{m} = 2 \quad or \ n = 2m. \ If \ m = 1,2,3.......etc., \ then \ n = 2,4,6.......etc$$

This means that the 2, 4, 6 etc. orders of interference maxima will be missing in the diffraction pattern. Thus the central diffraction maxima will have three interference maxima (the zero order and two first-orders).

If d=2b

$$\frac{b + 2b}{b} = \frac{n}{m} \quad or \ n = 3m. \ If \ m = 1,2,3.......etc., \qquad n = 3,6,9.......etc$$

This means that 3rd, 6th, 9th etc, orders of interference maxima will be missing in the diffraction pattern. On both sides of the central maximum, the number of interference maximum is 2 and hence there will be five interference maxima in the central diffraction maximum.