Diffraction, Classes of Diffraction & Fraunhofer Diffraction due to a Single Slit

Bp, Qsnpgw_ P_lg
Escqr D_asjrw* Bcn_rr kclr md Nfwqgaq*
K_e_bf K_fgj_ Amjjcec* NS
Ck_gj gb+ qsnpgw_,nfwqgaq>ek_gj,amk

Diffraction:

If an opaque obstacle is placed between a source of light and a screen then light bends around the corner of the obstacle into the geometrical shadow. This bending of light is called diffraction. The phenomenon of diffraction depends on the size of the obstacle and the wavelength of the light beam.

Diffraction is one particular type of wave interference, caused by the partial obstruction or lateral restriction of a wave. Not all interferences are diffraction; for example, sound waves emitted by two stereo speakers will interfere with each other if they are of the same frequency and have a definite phase relationship, but this is not diffraction. Diffraction will not occur if the wave is not coherent, and diffraction effects become weaker (and ultimately undetectable) as the size of obstruction is made larger and larger compared to the wavelength.

CLASSES OF DIFFRACTION

Based on the distance between source, aperture and screen, and also on the shape of wavefront, diffraction pattern is classified into two classes

1. **Fresnel Diffraction-**If the source of light and the screen are at finite distances from the diffracting aperture, then the wavefront falling on the aperture will not be plane (spherical or cylindrical). The diffraction obtained under this type of arrangement is called Fresnel Diffraction. This type of diffraction is also called near-field diffraction. No lenses are used to make the rays parallel or convergent.

   Fresnel Diffraction is obtained when light suffers diffraction at a straight edge, a thin wire, a narrow slit etc. Both the size and shape of the pattern depends on the distance between the diffracting aperture and the screen.

2. **Fraunhofer Diffraction-**If both the source of light and the screen are effectively far enough from the aperture so that the wavefronts reaching the aperture and the screen can be considered plane. Then the source and the screen are said to be at infinite distances from the aperture. This kind of diffraction is called Fraunhofer Diffraction. This is also called far-field diffraction.

Fraunhofer Diffraction is encountered in the case of gratings that contain number of slits. When the screen is moved, the size of the diffraction pattern changes uniformly while the shape of the pattern does not change.

FRAUNHOFER DIFFRACTION DUE TO A SINGLE SLIT

Let AB is a slit of width b, the diffracted beam through the slit is tilted at an angle θ with respect to straight direction.

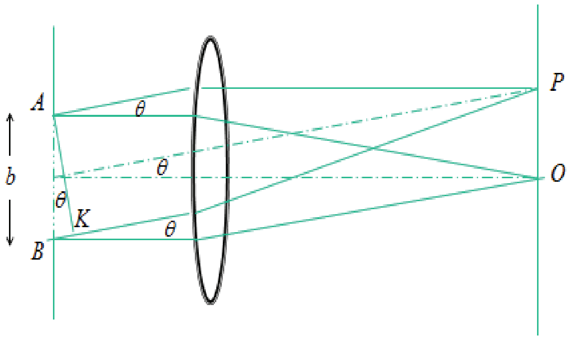

Figure1

Path difference between two rays diffracted from two extreme points of slit

$$= BK = AB\ sin\theta = b\ sin\theta$$

Phase difference $\quad = \dfrac{2\pi}{\lambda}$ x path difference $= \dfrac{2\pi}{\lambda}(b \sin\theta)$

Let the width AB of the slit be divide into n equal parts. The amplitude of vibration at P due to the waves from each part will be same, say *a*. The phase difference between the waves from any two consecutive parts is

$$\frac{1}{n}\left(\frac{2\pi}{\lambda} b \sin\theta\right) = 2\beta \text{ , say}$$

Then the resultant amplitude at P is given by

$$R = \frac{a \sin(nd/2)}{\sin(d/2)} = \frac{a \sin\left(\dfrac{\pi b \sin\theta}{\lambda}\right)}{\sin\left(\dfrac{\pi b \sin\theta}{n\lambda}\right)}$$

Let us put $\quad \left(\dfrac{\pi}{\lambda} b \sin\theta\right) = \alpha$

Then $\quad R = \dfrac{a \sin\alpha}{\sin(\alpha/n)} = \dfrac{a \sin\alpha}{\alpha/n} = \dfrac{na \sin\alpha}{\alpha} \quad$ .... (1)

When $n \to \infty$, $a \to 0$, but the product na remains finite.

Let $\quad na = A$

The resultant intensity at P, being proportional to the square of the amplitude, is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{....... (2)}$$

Condition for Maxima

$$R = \frac{A\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$R = \frac{A\sin\alpha}{\alpha} = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right] \qquad \text{....... (3)}$$

For $\alpha = 0$, $R = A$

This is the intensity of central maximum

$$\alpha = \left(\frac{\pi}{\lambda} b\sin\theta\right) = 0 \text{ or } \sin\theta = 0$$

Condition for Minima

$$\frac{\sin\alpha}{\alpha} = 0 \text{ or } \sin\alpha = 0, \text{ but } \alpha \neq 0$$

$\alpha = \pm m\pi$, Where m has an integral value 1, 2, 3 except zero

So $$\left(\frac{\pi}{\lambda} b\sin\theta\right) = \pm m\pi \Rightarrow b\sin\theta = \pm m\lambda \qquad \text{....... (4)}$$

This equation gives the position of first, second, third etc. minima for m = 1, 2, 3 etc

Secondary Maxima

$$\frac{dI}{d\alpha} = 0$$

or $$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

or $$A^2\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha = y \ (say)$$

$$y = \alpha \text{ and } y = \tan\alpha$$

The maxima will occur when

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

or $$\alpha = (2n+1)\frac{\pi}{2} \quad n = 1,2,3..... \qquad .... (5)$$

These are points of secondary maxima

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad .... (6)$$

Put $$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ etc.$$

$$I_1 = \frac{4}{9\pi^2}I_0,\ I_2 = \frac{4}{25\pi^2}I_0,\ I_3 = \frac{4}{49\pi^2}I_0 \ etc$$

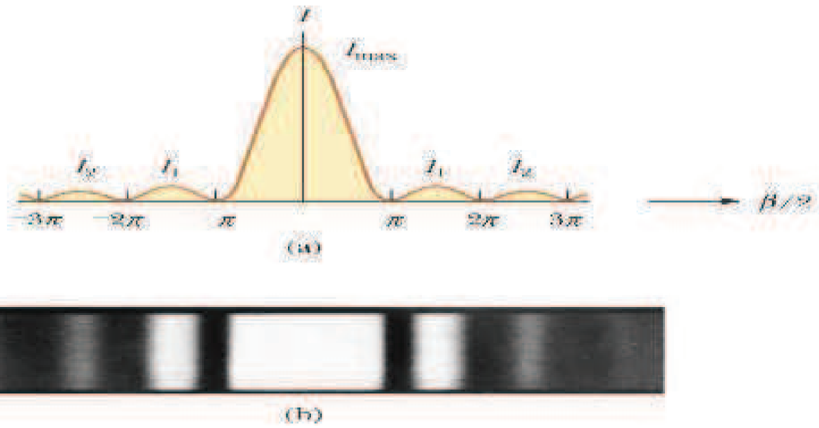

Figure 2