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Non-magnetic use of MHD equations : Ion-oscillations and waves

When the ions are displaced from their equilibrium position, the electrons adjust their positions in such a way that thermal equilibrium is maintained. Let an ion be displaced a distance ξ from its mean position; then the change in ion density as before,

$$\delta n^+ = n\frac{d\xi}{dx}. \quad \ldots(1)$$

where n is the number of ions per unit volume of plasma

The electrons adjusting themselves to equilibrium, received an amount of energy eV where V is the potential due to excess charge and hence according to Boltzman statistics the number of such particles is ne^{eV/KT_e}, where T_e is the electron temperature, then the change of electron density is $[ne^{eV/KT_e} - n]$

then $$\delta n_e = n\,[e^{eV/KT_e} - 1] \quad \ldots(2)$$

then from Poisson's equation

$$\begin{aligned}\frac{\partial E}{\partial x} &= -4\pi e\,[\delta n^+ - \delta n_e] \\ &= -4\pi e\left[n\frac{d\xi}{dx} - n\,e^{eV/KT_e} + n\right] \\ &= -4\pi n e\left[\frac{d\xi}{dx} - e^{eV/KT_e} + 1\right]. \quad \ldots(3)\end{aligned}$$

The equation of motion of mass M is

$$eE = M\frac{d^2\xi}{dt^2}$$

$$E = \frac{M}{e}\cdot\frac{d^2\xi}{dt^2}$$

$$e\frac{\partial^2 E}{\partial x^2} = M\frac{\partial^2}{\partial x^2}\cdot\frac{d^2\xi}{dt^2}.$$

$$\frac{\partial^2 E}{\partial x^2} = \frac{M}{e}\frac{\partial^2}{\partial x^2}\cdot\frac{d^2\xi}{dt^2}. \quad \ldots(4)$$

From **(3)** if eV/KT_e is << 1

$$\frac{\partial E}{\partial x} = -4\pi\, n\, e\left[\frac{d\xi}{dx} - 1 - \frac{eV}{KT_e} + 1\right]$$

$$= -4\pi\, n\, e\left[\frac{d\xi}{dx} - \frac{eV}{KT_e}\right]$$

$$\frac{\partial^2 E}{\partial x^2} = -4\pi\, n\, e\left[\frac{\partial^2 \xi}{\partial x^2} - \frac{eE}{KT_e}\right]. \qquad \ldots(5)$$

From equ. (4) and (5)

$$-4\pi\, n\, e\left[\frac{\partial^2 \xi}{\partial x^2} - \frac{eE}{KT_e}\right] = \frac{M}{e}\frac{\partial^2}{\partial x^2}\cdot\frac{d^2\xi}{dt^2}.$$

$$-\frac{4\pi\, n\, e^2}{M}\left[\frac{d^2\xi}{dx^2} - \frac{e}{KT_e}E\right] = \frac{\partial^2}{\partial x^2}\cdot\frac{d^2\xi}{dt^2}$$

then $$\frac{\partial^2}{\partial x^2}\left[\frac{d^2\xi}{dt^2} + \frac{4\pi\, e^2\, n}{M}\xi\right] - \frac{4\pi\, n\, e^3}{M.KT_e}E = 0$$

again $$eE = M\frac{d^2\xi}{dt^2}$$

$$\frac{\partial^2}{\partial x^2}\left[\frac{d^2\xi}{dx^2} + \frac{4\pi\, e^2\, n}{M}\xi\right] - \frac{4\pi\, n\, e^2}{KT_e}\cdot\frac{d^2\xi}{dt^2} = 0. \qquad \ldots\ \textbf{(6)}$$

Let $$\xi = A\cos\left(2\pi ft - \frac{2\pi x}{\lambda}\right)$$

$$\frac{d^2\xi}{dt^2} = -4\pi^2 f^2 A\cos\left(2\pi ft - \frac{2\pi x}{\lambda}\right)$$

$$\frac{\partial^2}{\partial x^2}\cdot\frac{d^2\xi}{dt^2} = \frac{16\pi^4 f^2}{\lambda^2}A\cos\left(2\pi ft - \frac{2\pi x}{\lambda}\right)$$

then from **(6)**

$$\frac{16\pi^4 f^2}{\lambda^2}A\cos\left(2\pi ft - \frac{2\pi x}{\lambda}\right) - \frac{16\pi^3\, n\, e^2}{\lambda^2 M}A\cos\left(2\pi ft - \frac{2\pi x}{\lambda}\right) + \frac{16\pi^3\, n\, e^2 f^2}{KT_e}A\cos\left(2\pi ft - \frac{2\pi x}{\lambda}\right) = 0$$

$$\frac{16\pi^4 f^2}{\lambda^2} - \frac{16\pi^3\, n\, e^2}{\lambda^2 M} + \frac{16\pi^3\, n\, e^2 f^2}{KT_e} = 0$$

$$\frac{\pi f^2}{\lambda^2} + \frac{n\, e^2 f^2}{KT_e} = \frac{n\, e^2}{\lambda^2 M}$$

$$f^2\left[\frac{\pi}{\lambda^2} + \frac{ne^2}{KT_e}\right] = \frac{ne^2}{\lambda^2 M}$$

$$f^2 = \frac{ne^2}{\lambda^2 M\left[\dfrac{\pi}{\lambda^2} + \dfrac{ne^2}{KT_e}\right]}$$

then $$f^2 = \frac{f_i^2}{1 + f_i^2 \frac{M\lambda^2}{KT_e}} \qquad \text{.... (7)}$$

$$\text{If } f_i = \sqrt{\frac{ne^2}{M\pi}} .$$

The actual frequency thus depends upon the wavelength. When λ is small, so that the first term in the denomi.iator is large the ion plasma oscillations are analogous to those of electrons having a frequency

$$f^2 = \frac{ne^2}{M\pi} .$$

This also holds for high electron temperature when λ is large and the second term predominates the oscillations become analogous to sound waves

i.e. $$f^2 = \frac{KT_e}{M\lambda^2}$$

then $$f^2\lambda^2 = \frac{KT_e}{M}$$

or $$f\lambda = v = \sqrt{\frac{KT_e}{M}} .$$

It should be noted that consideration of random thermal motion has resulted in a progressive wave as distinct from a stationary oscillation. Both the ion and electron oscillations discussed so far are pure electrostatic effects, the field and current fluctuations being in the direction of propagation and not perpendicular as in the case of e.m. waves. The transition from stationary ion oscillation to progressive ion waves occurs at wavelengths which satisfy

Since $$\frac{ne^2}{M\pi} = \frac{KT_e}{M\lambda^2}$$

$$\pi M = \frac{ne^2 M\lambda^2}{KT_e}$$

or $$\lambda = \sqrt{\frac{\pi KT_e}{ne^2}} \approx 2\sqrt{2}\,\lambda_d$$

where λ_d is the Debye shielding distance.