Velocity Transformation: B.Sc. Part-1, Hons. & Sub.

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VELOCITY TRANSFORMATION

Let us consider two inertial systems S and S' moving with relative velocity v along the xx'-axes. Let a particle at P which is moving with a velocity u as measured by an observer in S. Its velocity as measured by an observer in S' is u'. The velocity components in S and S' are

$$u_x = \frac{dx}{dt} \qquad u_y = \frac{dy}{dt} \qquad u_z = \frac{dz}{dt} \tag{1}$$

$$u'_x = \frac{dx'}{dt'} \qquad u'_y = \frac{dy'}{dt'} \qquad u'_z = \frac{dz'}{dt'} \tag{2}$$

Differentiation of the Lorentz transformation equation denoted by Eq. 1 gives:

$$dx' = \gamma(dx - \mathrm{v}dt) \qquad dy' = dy$$

$$dz' = dz \qquad dt' = \gamma\left(dt - \frac{\mathrm{v}dx}{c^2}\right) \tag{3}$$

Substitution of these values in Eq. 2

$$u'_x = \frac{dx - \mathrm{v}dt}{dt - (\mathrm{v}dx/c^2)} = \frac{(dx/dt) - \mathrm{v}}{1 - (\mathrm{v}/c^2)(dx/dt)}$$

$$u'_x = \frac{u_x - \mathrm{v}}{1 - (\mathrm{v}u_x/c^2)} \tag{4a}$$

$$u'_y = \frac{u_y}{\gamma(1 - \mathrm{v}u_x/c^2)} = \frac{u_y\sqrt{1-\beta^2}}{1 - (\mathrm{v}u_x/c^2)} \tag{4b}$$

$$u'_z = \frac{u_z}{\gamma(1 - \mathrm{v}u_x/c^2)} = \frac{u_z\sqrt{1-\beta^2}}{1 - (\mathrm{v}u_x/c^2)} \tag{4c}$$

These are the **Lorentz velocity transformations**. It may be noted that the velocity components u'_y and u'_z also depend on u_x. The inverse transformation is obtained by replacing v by –v and interchanging primed and unprimed co-ordinates:

$$u_y = \frac{u'_y}{\gamma[1+(\mathrm{v}u'_x/c^2)]} = \frac{u'_y\sqrt{1-\beta^2}}{1+(\mathrm{v}u'_x/c^2)} \qquad (5)$$

$$u_z = \frac{u'_z}{\gamma\,[1+(\mathrm{v}u'_x/c^2)]} = \frac{u'_z\sqrt{1-\beta^2}}{1+(\mathrm{v}u'_x/c^2)} \qquad (6)$$

If $\boldsymbol{u}'$ is along the x'-axis, $u'_x = u', u'_y = 0, u'_z = 0$. Then

$$u' = \frac{u-\mathrm{v}}{1-(u\mathrm{v}/c^2)} \quad \text{or} \quad u = \frac{u'+\mathrm{v}}{1+(u'\mathrm{v}/c^2)} \qquad (7)$$

This Equation is referred to as *Einstein's law of addition of velocities*. Here, v is the velocity of frame S' with respect to S and u' is the velocity of the event P relative to S' and u is the velocity of the event P relative to S. As shown in figure

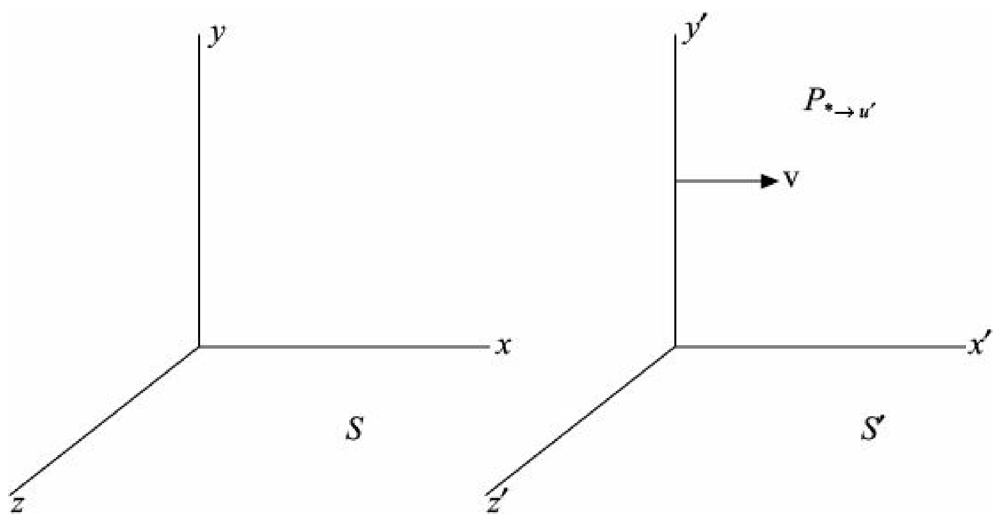

Fig. Inertial frames of reference S, S' and event at P.

If $u' = c$, the velocity of light

$$u = \frac{c+\mathrm{v}}{1+\frac{\mathrm{v}}{c}} = c$$

That is, the velocity of the source does not add anything to the velocity of light emitted by it. And we may say that, it is impossible to exceed the velocity of light by adding two or more velocities, no matter how close each of these velocities to are that of light.