LORENTZ TRANSFORMATION: B.Sc. Part-1, Hons. & Sub.

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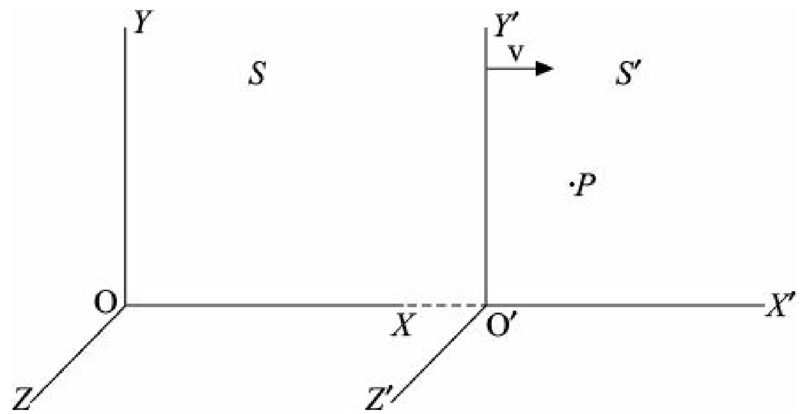

Fig. 1 The inertial system S and S' with co-ordinates axes xyz and $x'y'z'$.

Let us consider two reference frames S and S' moving with uniform relative motion . Let two observers O and O' observe any event P from systems S and S', respectively. Let the event P is produced at $t = 0$ when the origins of the two frames coincide. For the observer at O, the co-ordinates of the event at a particular instant be (x, y, z, t). The same event is described by the co-ordinates (x', y', z', t') for the observer O' on the system S'.

The velocity of S' with respect to S is along the x-axis. Hence,

$$y' = y \text{ and } z' = z \qquad (1)$$

A uniform rectilinear motion in S must go over into a uniform rectilinear motion in S'. Hence, the transformation relating x and x' must be linear. A non-linear transformation may produce acceleration in S' even if the velocity is constant in S. In addition, the transformation must reduce to Galilean transformation at low speeds. Therefore, the transformation equation relating x and x' can be written as

$$x' = k(x - \mathrm{v}t) \qquad (2)$$

where k is independent of x and t. Since S is moving relative to S' with velocity **v** along the positive x-axis,

$$x = k(x' + \mathrm{v}t') \qquad (3)$$

The same constant k is used, since nothing distinguishes S and S' from one another except the sign of the relative velocity. Substituting the value of x' from Eq. (2), we have

$$x = k\,[k(x - \mathrm{v}t) + \mathrm{v}t']$$

or,

$$t' = kt + \frac{(1-k^2)}{k\mathrm{v}}x \qquad (4)$$

We then write the the transformation equations as:

$$\begin{aligned} x &= k(x - \mathrm{v}t) \\ y' &= y \\ z' &= z \\ t' &= kt + \frac{(1-k^2)x}{k\mathrm{v}} \end{aligned} \qquad (5)$$

Let us assume that the event is a pulse of light emitted from O at time $t = 0$. The pulse of light spreads as a spherical wave travelling with the velocity c. The equation of the wavefront in S at time t is:

$$x^2 + y^2 + z^2 = c^2\,t^2 \qquad (6)$$

The wavefront in the reference frame S' is

$$x'^2 + y'^2 + z'^2 = c^2\,t'^2 \qquad (7)$$

Here, we know that the velocity of the light wave is the same in all directions in either frame of reference. Substituting the tentative transformation equations, Eq. (5) in Eq. (7), we have

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$$k^2\,(x - \mathrm{v}t)^2 + y^2 + z^2 = c^2k^2t^2 + \frac{2c^2tx\,(1-k^2)}{\mathrm{v}} + \frac{c^2x^2\,(1-k^2)^2}{k^2\mathrm{v}^2} \qquad (8)$$

We must choose k such that Eq. (8) reduces to Eq. (6), since each equation represents the position of the wavefront as measured in *S*. Comparing the coefficients of the terms in *x* in both equations

$$2k^2 vt + \frac{2c^2 t(1-k^2)}{v} = 0$$

$$k = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \textbf{(9)}$$

k gives the co-ordinate transformation equations:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad \textbf{(10 a)}$$

$$y' = y \qquad \textbf{(10 b)}$$

$$z' = z \qquad \textbf{(10 c)}$$

$$t' = \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}} \qquad \textbf{(10 d)}$$

Equation (10) is called the **Lorentz transformation** equations.

The inverse transformation can be obtained by interchanging the primed and unprimed quantities and reversing the sign of the relative velocity, since *S* and *S*′ differ only in the sign of the relative velocity.

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} \qquad \textbf{(11)}$$

the Lorentz transformation and the inverse transformation simplify to

$$x' = \gamma(x - \beta ct) \qquad x = \gamma(x' + \beta ct') \qquad \textbf{(12 a)}$$

$$y' = y \qquad y = y' \qquad \textbf{(12 b)}$$

$$z' = z \qquad \text{or} \qquad z = z' \qquad \textbf{(12 c)}$$

$$t' = \gamma\left(t - \frac{\beta}{c}x\right) \qquad t = \gamma\left(t' + \frac{\beta}{c}x'\right) \qquad \textbf{(12 d)}$$

In the low velocity limit, where *b* << 1, it follows that the Lorentz transformation reduces to the Galilean transformation. The equation is sometimes referred to as **space-time transformation.**

Lorentz transformation sets a limit on the maximum value of v. If v > *c*, the quantity $\sqrt{1-\beta^2}$ becomes imaginary. The space and time co-ordinates would then become imaginary, which is physically unacceptable. Hence, in vacuum nothing can move with a velocity greater than the velocity of light.