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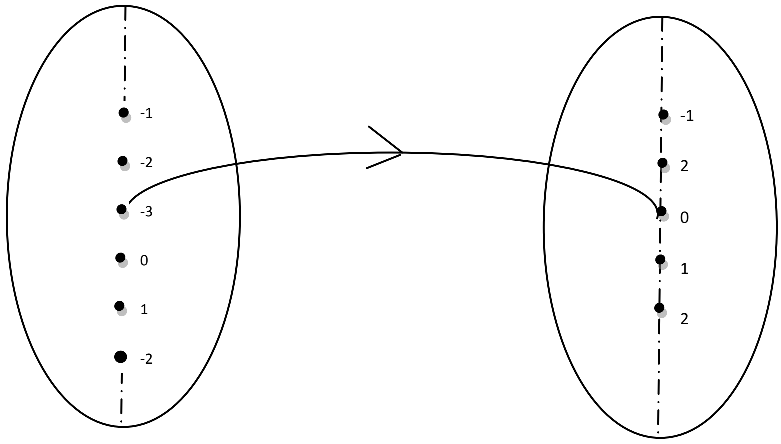
Program for B.Sc (Hons) Part-3

Some Standard Exemplas of Automorphism

Definition :- An isomorphism from a group G onto itself is called Automorphism of G.

❖ The set of all automorphsm of a group G is denoted by Aut (G).

Ex:- The mapping ϕ :- $(\mathbb{Z}, +)\to(\mathbb{Z}, +)$ defined by $\phi(x)=2x \ \forall\ x \in \mathbb{Z}$ is not an automorphism as the mapping not onto.

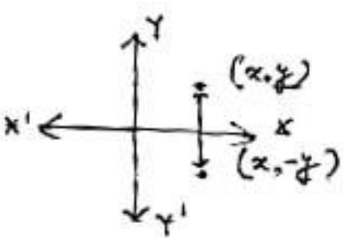


Ex :- Let the mapping $\phi_K : (\mathbb{Z}_n, \oplus_n)\to(\mathbb{Z}, \oplus_n)$ be defined by $x\to K_n(\text{mod } x)$ where $(k,n)= 1$. Then ϕ_K is an automorphism.

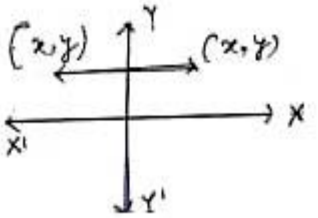
Some standard automorphism

The following mapping are automorphisms of $(IR^2, +)$:-

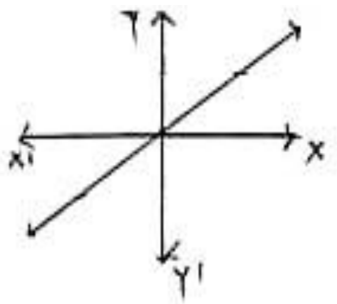
(i) $(x,y)\to(x\text{-}y)$ i.e. refection on X-axis



(ii) $(x,y) \to (-x,y)$ i.e. reflection on Y-axis



(iii) $(x,y) \to (y,x)$ i.e. reflection about $y=x$

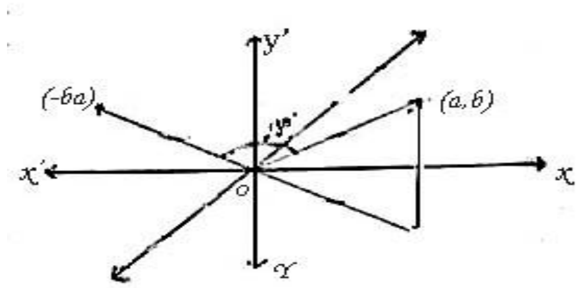


(iv) $(x,y) \to (cx,cy)$ where 'c' is a fixed non-zero real number

contraction $o<c<1$

Dilation if $c>1$.

Ex :- Consider a mapping ϕ defined on $(IR^2, +)$ by ϕ ([a, b])= (-b, a)



Clearly ϕ is objective.

Let x = (a, b) , y= (c, d)

Now, we have to prove that

$$\phi(x+y) = \phi(x)+(y)$$

$$\Rightarrow \phi(x+y) = \phi[(a,b)+(c,d)]$$

$$= \phi[a+c, b+d]$$

$$= [-b-d, a+c]$$

$$= (-b,a)+(-d,c)$$

$$= \phi(a,b)+\phi(y)(=\phi(c,d))$$

$= \phi(x) + \phi(y)$

So that ϕ preserve the operation.

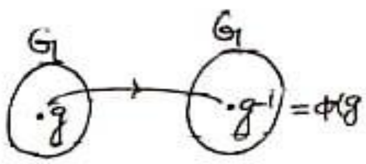
Hence ϕ is an automorphism.

Theorem: - Let G be a group and $\phi: G \rightarrow G$ be defined by $\phi(g) = g^{-1}$ ∀ $g \in G$ then ϕ is an automorphism of G iff G is an abelian

Proof :- Let ϕ is an automorphism . This implies that g, h ϵ G,

$\phi(gh) = \phi(g).\ \phi(h)$

$(gh)^{-1} = h^{-1}g^{-1}$



But, by the definition of function for all g,h,ϵG

$$(gh)^{-1} = g^{-1}h^{-1}$$

So, $g^{-1}h^{-1} = h^{-1}g^{-1}$

$\Rightarrow (g^{-1}h^{-1})^{-1} = (h^{-1}g^{-1})^{-1}$

$\Rightarrow hg = gh$ ∀ $g, h, \in G$

$\Rightarrow$ G is abelian.

Conversely, let G is abelian.

$\Rightarrow gh = hg = gh$ ∀ $g, h \in G$

And $h^{-1}g^{-1} = g^{-1}h^{-1}$

Now, ∵ $\phi(g) = g^{-1} \Rightarrow \phi(gh) = g^{-1}h^{-1} = h^{-1}g^{-1}$

$= \phi(h).\ g(h)$

So, ϕ is operation preserving,. Hence ϕ is an automorphism.