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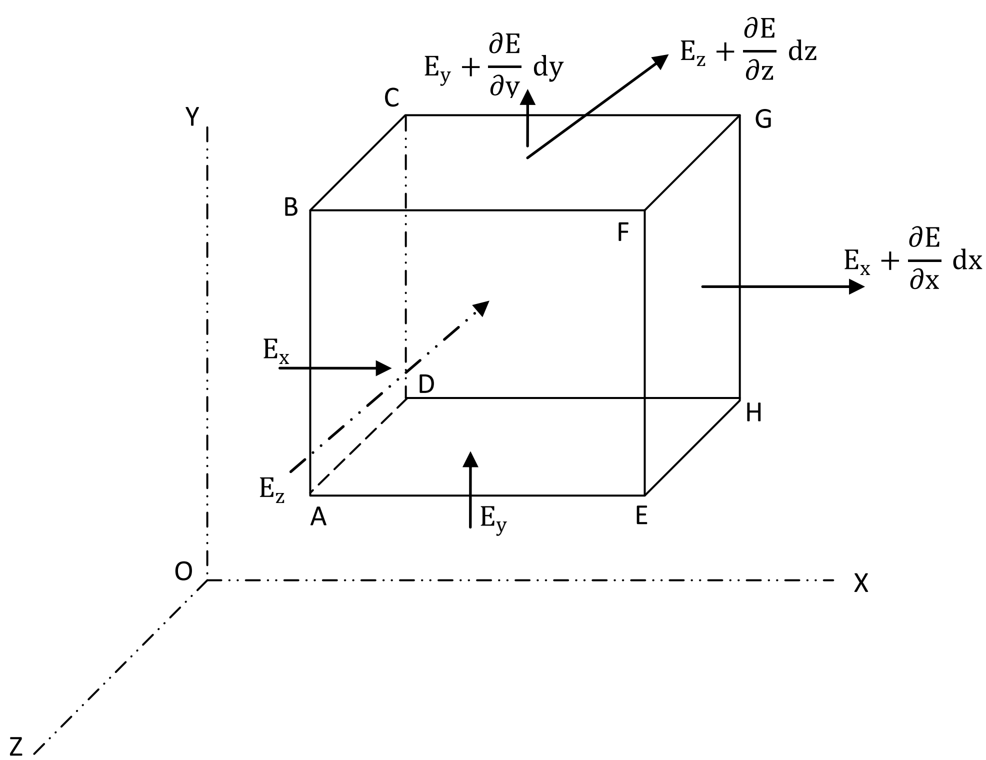
Poisson's and Laplace's equation for an electric field :-



Consider a rectangular parallelopiped (ie. Cuboid) whose sides are of infinitesimal length ∂x , ∂y. ∂z

K=Dielectric constant of medium immerssed

E = Electric intensity at A

Electric field at face ABCD=E_x

Electric field at face EFGH=E_x+$\frac{\partial E_x}{\partial x}$

Normal induction over face ABCD=ε_0 KE_x dydz (directed inward)

Normal induction over face EFGH=ε_0 K $\left(E_x + \frac{\partial E_x}{\partial x}\ dx\right)$dydz (directed outward)

Contribution of total normal electric induction of faces ABCD & EFGH

$$= \varepsilon_0 K \left(E_x + \frac{\partial E_x}{\partial x} dx\right) dy\, dz - \varepsilon_0 K E_x\, dy\, dz$$

$$= \varepsilon_0 K \frac{\partial E_x}{\partial x} dx\, dy\, dz$$

Similarly

Contribution of total normal electric induction of faces ADHE & BCGF

$$= \varepsilon_0 K \frac{\partial E_y}{\partial y} dx\, dy\, dz$$

Contribution of total normal electric inductiom of faces ABFE & DCGH

$$= \varepsilon_0 K \frac{\partial E_z}{\partial z} dx\, dy\, dz$$

Total outward N.E.I or normal electric flux over the parallelopiped surface

$$= \varepsilon_0 K \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) dx\, dy\, dz$$

If ρ = volume charge density

By Gauss's theoram

$$= \varepsilon_0 K \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) dx\, dy\, dz = \rho.\, dx\, dy\, dz$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0 K}$$

This is Poisson's equation

$$\text{div } E = \frac{\rho}{\varepsilon_0 K}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0 K}$$

$\vec{E} = \text{Electric vector}$

$\because E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$

$\because$ V = Potential at point A

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0 K}$$

div. grad V = $-\frac{\rho}{\varepsilon_0 K}$

$$\nabla^2 . V = -\frac{\rho}{\varepsilon_0 K}$$

When $\rho = 0$ ie. There are no charges in the field

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\nabla \cdot \vec{E} = O$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = O$$

$$\nabla^2 V = O$$

This is Laplace's equation