NumericalProblems & Solutions Based on Concept of Lorentz Transformation: B.Sc. Part-1 Hons.

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Question1 Show that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz Transformation

Solution: Replacing x, y, z, t by x', y', z', t'

$$x^2 + y^2 + z^2 - c^2t^2 = \gamma^2 (x' + vt')^2 + y'^2 + z'^2 - c^2\gamma^2\left[t' + \left(vx'/c^2\right)\right]^2$$

$$= x'^2\gamma^2\left(1 - \frac{v^2}{c^2}\right) + y'^2 + z'^2 - t'^2\gamma^2c^2\left(1 - \frac{v^2}{c^2}\right)$$

$$= x'^2 + y'^2 + z'^2 - c^2t'^2$$

That is, $x^2 + y^2 + z^2 - c^2 t^2$ is invariant under Lorentz transformation.

Question2: A rocket leaves the earth at a speed of 0.6c. A second rocket leaves the first at a speed of 0.9c with respect to the first. Calculate the speed of the second rocket with respect to earth if: (i) it is fired in the same direction as the first one; (ii) it is fired in a direction opposite to the first.

Solution: (i) $u = \dfrac{u' + v}{1 + \dfrac{u'v}{c^2}} = \dfrac{0.9c + 0.6c}{1 + \dfrac{(0.9c)\ (0.6c)}{c^2}} = 0.974c$

(ii) $u = \dfrac{-0.9c + 0.6c}{1 - \dfrac{(0.9c)\,(0.6c)}{c^2}} = -\dfrac{0.3c}{0.46} = -0.652c$

Question:3 The length of a spaceship is measured to be exactly half its proper length. What is (i) the speed of the spaceship relative to the observer on earth? (ii) the dilation of the spaceship's unit time?

Solution: (i) Taking the spaceship's frame as the S' one, the length in the frame S is given by

$$L = L_0 \sqrt{1-\beta^2} \qquad \beta = \mathrm{v}/c$$

It is given that $L = L_0/2$. Then

$$\frac{L_0}{2} = L_0\sqrt{1-\frac{\mathrm{v}^2}{c^2}} \qquad \text{or} \qquad 1-\frac{\mathrm{v}^2}{c^2} = \frac{1}{4}$$

$$\mathrm{v} = 0.866\ \mathrm{c}$$

(ii) From] ive

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\mathrm{v}^2/c^2}} = \frac{\Delta\tau}{1/2} = 2\Delta\tau$$

That is, unit time in the S' clock is recorded as twice of unit time by the observer. Or we can say, the spaceship's clock runs half as fast.

Question:4 An inertial frame S' moves with respect to another inertial frame

S with a uniform velocity 0.6 c along the x x'-axes. The origins of the two systems coincide when $t = t' = 0$. An event occurs at $x_1 = 10$ m, $y_1 = 0$, $z_1 = 0$, $t_1 =$

2×10^{-7}s. Another event occurs at $x_2 = 40$m, $y_2 = 0$, $z_2 = 0$, $t_2 =$

3×10^{-7}s . In S' , (i) what is the time difference? (ii) what is the distance between the events?

Solution: (i) We know

$$t_2' - t_1' = \gamma(t_2 - t_1) + \frac{\gamma \mathrm{v}}{c^2}(x_1 - x_2)$$

$$\gamma = \frac{1}{\sqrt{1-\mathrm{v}^2/c^2}} = \frac{1}{\sqrt{1-0.36}} = \frac{1}{\sqrt{0.64}} = 1.25$$

$$t_2' - t_1' = 1.25 \times (3-2)10^{-7}\,\mathrm{s} + \frac{1.25 \times 0.6\,(-30\mathrm{m})}{3 \times 10^8\,\mathrm{ms}^{-1}}$$

$$= 1.25 \times 10^{-7}\,\mathrm{s} - 0.75 \times 10^{-7}\mathrm{s} = 0.5 \times 10^{-7}\mathrm{s}$$

(ii) From Lorentz transformation

$$x_1' = \gamma\left(x_1 - \mathrm{v}t_1\right) \qquad x_2' = \gamma\left(x_2 - \mathrm{v}t_2\right)$$

$$x_2' - x_1' = \gamma\left(x_2 - x_1\right) - \gamma\mathrm{v}\left(t_2 - t_1\right)$$

$$= 1.25 \times 30\,\mathrm{m} - 1.25(0.6)\left(3 \times 10^8\,\mathrm{ms^{-1}}\right)10^{-7}\,\mathrm{s}$$

$$= 37.5\,\mathrm{m} - 22.5\,\mathrm{m} = 15\,\mathrm{m}$$