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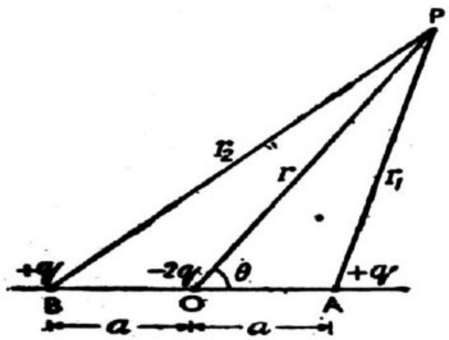
Electric potential at a Point due to an Electric Quadrupole



A quadrupole consists of two electric dipoles placed end to end along the same line. OA and OB are the two dipoles. The system AOB is called a quadrupole. The charge at $A = +q$, the charge at $B = +q$ and the charge at $O = -2q$. The total charge of the system as a whole is zero.

The electric potential at the point P,

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} - \frac{2q}{r} + \frac{q}{r_2}\right] \qquad ...(i)$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right]$$

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

$$r_1 = [r^2 - 2ar\cos\theta + a^2]^{\frac{1}{2}}$$

$$r_1 = r\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)^{\frac{1}{2}}$$

or $$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)^{-\frac{1}{2}}$$

Expanding the right hand side

$$\frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{1}{2}\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right) + \frac{3}{8}\left(-\frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)^2 + \dots\right]$$

Simplifying and retaining only the terms with r^3 or less in the denominator,

$$\frac{1}{r_1} = \frac{1}{r} + \frac{a\cos\theta}{r^2} + \frac{a^2}{2r^3}(3\cos^2\theta - 1) + \dots \qquad (ii)$$

Similarly, $r_2^2 = r^2 + a^2 + 2ar\cos\theta$, and proceeding as above,

$$\frac{1}{r_2} = \frac{1}{r} - \frac{a\cos\theta}{r^2} + \frac{a^2}{2r^3}(3\cos^2\theta - 1) \qquad ...(iii)$$

Substituting the values of $\frac{1}{r_1}$ and $\frac{1}{r_2}$ in equation (i)

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{a\cos\theta}{r^2} + \frac{a^2(3\cos^2\theta - 1)}{2r^3} + \frac{1}{r} - \frac{a\cos\theta}{r^2} + \frac{a^2}{2r^3}(3\cos^2\theta - 1) - \frac{2}{r}\right]$$

$$\therefore \quad V = \frac{q}{4\pi\epsilon_0}\left[\frac{a^2(3\cos^2\theta - 1)}{r^3}\right]$$