Coriolis Force: B.Sc. Part-1, Hons. & Sub.

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Coriolis Force

Rotational motion of rigid bodies is generally formulated in a body-fixed co-ordinate system. To convert results from a body-fixed system to a space fixed- system and vice versa, we should know how the time derivative of a vector in one system changes to the time derivative in the other system. Let O*xyz* be the
co-ordinate system fixed to the rotating body and $x'y'z'$ be the space-fixed one with common origin. Let the unit vectors of the body fixed system be $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. The radius vector of a mass point at *P* of the body with respect to the body-fixed system is

$$\mathbf{r} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z \tag{1}$$

As unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are constants with respect to the body-fixed system, specified by the subscript r

$$\left(\frac{d\mathbf{r}}{dt}\right)_r = \hat{\mathbf{i}}\frac{dx}{dt} + \hat{\mathbf{j}}\frac{dy}{dt} + \hat{\mathbf{k}}\frac{dz}{dt} \tag{2}$$

When we consider the time derivative of $\mathbf{r}$ with respect to the space-fixed system, the unit vectors also possess time derivatives as they change directions. Hence,

$$\begin{aligned}\left(\frac{d\mathbf{r}}{dt}\right)_s &= \hat{\mathbf{i}}\ \frac{dx}{dt} + \hat{\mathbf{j}}\frac{dy}{dt} + \hat{\mathbf{k}}\frac{dz}{dt} + \frac{d\hat{\mathbf{i}}}{dt}x + \frac{d\hat{\mathbf{j}}}{dt}y + \frac{d\hat{\mathbf{k}}}{dt}z \\ &= \left(\frac{d\mathbf{r}}{dt}\right)_r + \frac{d\hat{\mathbf{i}}}{dt}x + \frac{d\hat{\mathbf{j}}}{dt}y + \frac{d\hat{\mathbf{k}}}{dt}z \end{aligned} \tag{3}$$

, we can write

$$\frac{d\hat{\mathbf{i}}}{dt}x = \boldsymbol{\omega} \times \hat{\mathbf{i}}x \qquad \frac{d\hat{\mathbf{j}}}{dt}y = \boldsymbol{\omega} \times \hat{\mathbf{j}}y \qquad \frac{d\hat{\mathbf{k}}}{dt}z = \boldsymbol{\omega} \times \hat{\mathbf{k}}z \tag{4}$$

we can write

$$\left(\frac{d\mathbf{r}}{dt}\right)_s = \left(\frac{d\mathbf{r}}{dt}\right)_r + \boldsymbol{\omega} \times (\hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z) \tag{5}$$

$$= \left(\frac{d\mathrm{r}}{dt}\right)_r + (\boldsymbol{\omega} \times \mathbf{r}) \tag{6}$$

Generalizing to a general vector $\mathbf{A}$

$$\left(\frac{d\mathbf{A}}{dt}\right)_s = \left(\frac{d\mathbf{A}}{dt}\right)_r + \boldsymbol{\omega} \times \mathbf{A} \tag{7}$$

From this, we get the important operator equation

$$\left(\frac{d}{dt}\right)_s = \left(\frac{d}{dt}\right)_r + \boldsymbol{\omega} \times \tag{8}$$

where w is the angular velocity vector of the rotating body.

$$\mathbf{v_s} = \mathbf{v_r} + \boldsymbol{\omega} \times \mathbf{r} \qquad (9)$$

Equations that the relation connecting the inertial acceleration of the particle of mass *m* at *P* and its acceleration relative to the rotating frame get the time rate of change of $\mathbf{v}_S$

$$\left(\frac{d\mathbf{v}_s}{dt}\right)_s = \left(\frac{d\mathbf{v}_s}{dt}\right)_r + \boldsymbol{\omega} \times \mathbf{v}_s \qquad (10)$$

Replacing $\mathbf{v}_s$ on the right side of Eq. (9)

$$\left(\frac{d\mathbf{v}_s}{dt}\right)_s = \left(\frac{d\mathbf{v}_r}{dt}\right)_r + \left(\frac{d(\boldsymbol{\omega} \times \mathbf{r})}{dt}\right)_r + \boldsymbol{\omega} \times \mathbf{v_r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad (11)$$

$$= \left(\frac{d\mathbf{v}_r}{dt}\right)_r + \left(\frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r}\right)_r + \boldsymbol{\omega} \times \mathbf{v_r} + \boldsymbol{\omega} \times \mathbf{v_r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad (12)$$

When angular velocity is constant, $(d\boldsymbol{\omega}/dt) = 0$. The factor $(d\mathbf{v}_s/dt)_s$ is the inertial acceleration $\mathbf{a}_s$ of the particle relative to the inertial system, and $(d\mathbf{v}_r/dt)_r$ is the acceleration $\mathbf{a}_r$ of the particle relative to the rotating co-ordinate system. Then

$$\mathbf{a}_s = \mathbf{a}_r + 2(\boldsymbol{\omega} \times \mathbf{v}_r) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad (13)$$

The equation of motion in the inertial system is

$$\mathbf{F} = m\mathbf{a}_s \quad (14)$$

Multiplying eq (13)) by m and replacing $m\,\mathbf{a}_s$ by $\mathbf{F}$

$$\mathbf{F} - 2m(\boldsymbol{\omega} \times \mathbf{v}_r) - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) = m\mathbf{a}_r \quad (15)$$

To an observer in the rotating system, it appears as if the particle is moving under the influence of an effective force

$$\mathbf{F}_{eff} = \mathbf{F} - 2m(\boldsymbol{\omega} \times \mathbf{v}_r) - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathrm{r}) \quad (16)$$

The third term on the right, $-m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$, is called the **centrifugal force**. Its magnitude

$$|\, m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \,| = mr\omega^2 \sin\theta \quad (17)$$

where q is the angle between vectors w and **r**. This reduces to $-mr$w^2 when w is normal to the radius vector (circular motion). The negative sign indicates that the centrifugal force is directed away from the centre of rotation. It is not a real force, but a fictitious one. It is present only if we refer to moving co-ordinates in space.

The second term $2m(\mathbf{w} \times \mathbf{v}_r)$, called the **Coriolis force**, is present when a particle is moving in the rotating co-ordinate system. This is also not a real force, but a fictitious one. It is directly proportional to $\mathbf{v}_r$ and will disappear when there is no motion. Another feature of this force is that it does no work, since it acts in a direction perpendicular to velocity.

The centrifugal and coriolis forces are not due to any physical interaction, and hence they are non-inertial or fictitious forces. The rotating earth can be considered a rotating frame. Though its angular velocity is small, it has considerable effect on some of the quantities. Some of them are:

(i) The Coriolis force has to be taken into account to compute accurately the trajectories of long range projectiles and missiles.

(ii) It is the Coriolis force on moving masses that produces a counterclockwise circulation in the northern hemisphere which affects the course of winds.

(iii) The spinning motion of the earth is that which causes the equatorial bulge.