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Plasma Oscillation Frequency

The first simple treatment of plasma oscillations was given by Tonks and Langmuir who considered an idealized plasma with no thermal motion of ions and electrons in which two types of oscillation are basically possible. First the electron oscillation which are so fast that ions can be regarded as stationary and secondly the ion oscillation which are so slow that electrons at all times adjust their energy and density so as to remain in equilibrium and satisfy Boltzmann distribution.

Suppose somewhere electors are accumulated instantaneously in a plasma in some region. They will produce electric field at that region. This electric field moves electrons away from there and creating a deficiency of electrons here. So electrons move out from here and leaving behind a positive charge there. This positive charge attracts electrons back and in this the electrons oscillate.

Let each electron be displaced a small distance ξ in the x direction only.

Change per unit distance = $\frac{d\xi}{dx}$

Change in charge density due to motion of electron

$$d\rho = n_e e \frac{d\xi}{dx}$$

Where n_e= number of electrons per unit volume i.e. electron density

As a result of this momentary disturbance of charge neutrality an electric field E develops which is given by Gauss's theorem

div E = 4πdρ

From Poisson's equation

$$\frac{d^2v}{dx^2} = \frac{dE}{dx} = 4\pi\ ne\frac{d\xi}{dx}$$

Integrating both side

∴ E = 4π ne ξ + constant

As there is no externally applied field

constant =O

This field produces restoring force and hence

$$m\frac{d^2\xi}{dt^2} = -e \in = -4\pi ne^2\xi$$

$$\frac{d^2\xi}{dt^2} = \frac{-4\pi n e^2\xi}{m}$$

Which is an oscillatory motion of the electron with an angular frequency

$$\omega_p^2 = \frac{4\pi n e^2}{m}$$

$$\omega_p = \sqrt{\frac{4\pi n e^2}{m}}$$

$$f_p = \sqrt{\frac{ne^2}{m\pi}} = 8980\sqrt{n}$$

ω_p is called electron plasma frequency and from the generalized theory of ionosphere, it is the critical frequency below which the incident waves are reflected from the plasma.

It is know that in a gas any particle displacement leads to a pressure disturbance which propagates at the speed of sound wave.

The motion of the electrons with respect to ions which are assumed to be stationary will not only give rise to an electric field but also produce a pressure wave.

Consider the pressure wave in case of acoustics

$$\rho_o \frac{d^2\xi}{dt^2} = \frac{\partial P}{\partial x}$$

where P is the excess pressure

$$P = - \beta\frac{d^2\xi}{dt^2}$$

where β is the adiabatic elasticity of gas

$$\beta = rP$$

where r is the ratio of specific heats of gas

$$\rho_o \frac{d^2\xi}{dt^2} = rP\frac{d^2\xi}{dx^2}$$

when the force due to charge reparation is taken into account.

$$\rho_o \frac{d^2\xi}{dt^2} = -4\pi n^2 e^2 \xi + rP\frac{d^2\xi}{dx^2}$$

$$\rho_o \frac{d^2\xi}{dt^2} + 4\pi n^2 e^2 \xi = rP\frac{d^2\xi}{dx^2}$$

$$\frac{d^2\xi}{dt^2} + \frac{4\pi n^2 e^2 \xi}{\rho_o} = \frac{rP}{\rho_o}\frac{d^2\xi}{dx^2}$$

$$\frac{d^2\xi}{dt^2} + \frac{4\pi n^2 e^2 \xi}{nm} = V_s^2\frac{d^2\xi}{dx^2}$$

Where V_s is the velocity of sound through electron gas

$$\frac{d^2\xi}{dt^2} + \omega_p^2\xi = V_s^2\frac{d^2\xi}{dx^2}$$

where ω_p is the electron plasma frequency

Let $\xi = Ae^{j\,(kx-\omega t)}$

Where K is the wave vector, $k = {}^{2\pi}/_{\lambda}$

$$-\,\omega^2\xi + \omega_p^2 = -\,V_s^2k^2\xi$$

$$\omega^2 = \omega_p^2 + V_s^2k^2$$

$$\omega^2 = \omega_p^2 + V_s^2\frac{4\pi^2}{\lambda^2}$$

$$f^2 = f_p^2 + \frac{v_x^2}{\lambda^2}$$

This equation on which shows the frequency as function of velocity is known as dispersion relation. The phase velocity is

$$V_p = \frac{\omega}{k}$$

$$V_p = \left[\frac{\omega_p^2}{k^2} + V_s^2\right]^{\frac{1}{2}}$$

The phase velocity of electron oscillation is always greater than the acoustic velocity. For very long wavelength of frequency of oscillation becomes equal to plasma frequency and phase velocity tends to infinity which means that entire plasma volume oscillates with a constant plasma frequency.