# Thomson and Rayleigh Scattering of Electromagnetic Waves

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The *differential scattering cross-section* is given by:

$$\sigma(\Omega) = \frac{r_0^2\omega^4}{\left[(\omega_0^2 - \omega^2)^2 + g^2\omega^2\right]}\left\{\frac{1}{2}(1+\cos^2\phi)\right\} \tag{1}$$

The total *scattering cross-section* is given by:

$$\sigma_T = \frac{8}{3}\pi\frac{r_0^2\omega^4}{\left[(\omega_0^2 - \omega^2)^2 + g^2\omega^2\right]} \tag{2}$$

Let us consider the particular cases of scattering:

## 1 Case (i) Scattering by a free electron: Thomson scattering

When the electron is free $\omega \gg \omega_0$ (since $K \longrightarrow 0$) and damping constant $g = 0$, therefore equations (1) and (2) for differential scattering cross-section and total scattering cross-section take the form

$$\sigma(\Omega) = r_0^2\frac{1}{2}(1+\cos^2\phi) \tag{3}$$

and

$$\sigma_T = \frac{8}{3}\pi r_0^2 \tag{4}$$

These equation represents *Thomson Scattering formulae* for differential and total scattering cross-sections. These formulae apply only to free electrons and are appropriate for scattering of X-rays by electrons or $\gamma$-rays by protons. From these formulae it is clear that

1. Thomson scattering of electromagnetic waves is independent of frequency or wavelength of incident waves,

2. Scattering depends on the classical radius of electron i.e. the nature of scatterer.

3. Scattering occurs in all directions and is maximum for $\phi = 0$ and $\pi$ (i.e. in the forward and backward directions) while it is minimum for $\phi = \pi/2$ or $3\pi/2$ (i.e. in the side way directions). Moreover scattering is symmetrical about the line $\phi = \pi/2$.

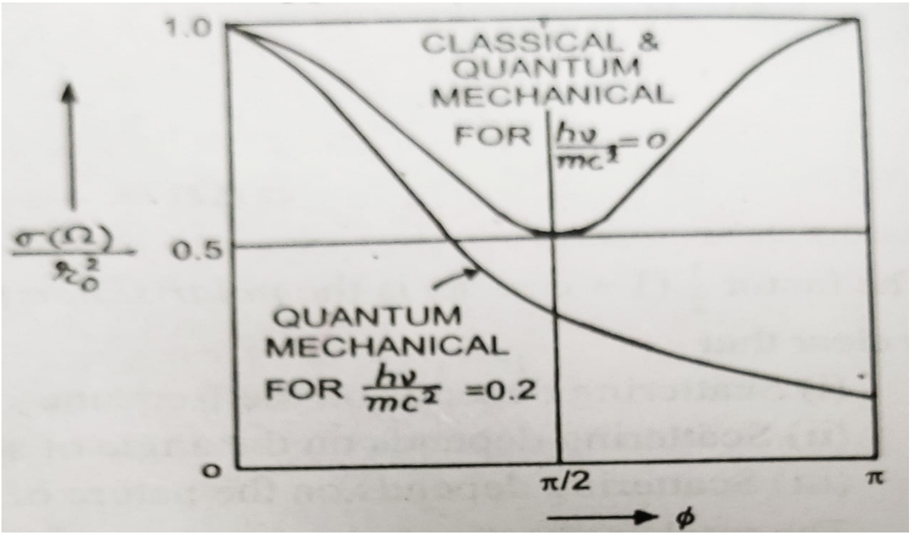


Figure 1:

Fig. (1) represents the plot of differential cross-section against scattering angle $\phi$ according to classical and quantum mechanical theories.The quantum mechanical formula approaches Thomson formula as the frequency goes to zero.

## 2 Case (ii) Resonant Scattering

When $\omega = \omega_0$, then equation (2) gives

$$\sigma_T = \frac{8}{3}\pi r_0^2 \left(\frac{\omega_0}{\omega}\right)^2 \tag{5}$$

In this case the total scattering cross-section is maximum, thus resulting in very large scattering known as *resonant scattering.* The example of resonant scattering is that a bulb filled with sodium vapour which has a natural frequency in the visible region, illuminated with light of this colour, will scatter so much that it appears luminous.

## 3 Case (iii) Rayleigh Scattering

If $\omega \ll \omega_0$, then

$$\sigma_T = \frac{8}{3}\pi r_0^2 \left(\frac{\omega}{\omega_0}\right)^4 \tag{6}$$

*The scattering is proportional to $\omega^4$ i.e. fourth power of frequency of incident radiation or inversely proportional to the fourth power of wavelength of incident radiation (i.e. $\sigma_T \propto 1/\lambda^4$).* The scattering is known as *Rayleigh scattering.* This is likely to occur when $\omega$ corresponds to frequencies of visible light and $\omega_0$ to that of ultraviolet.

Rayleigh scattering is capable of explaining the phenomenon like blue colour of sky, red colour of sunrise or sunset, use of red light for danger signals too.

## 4 Blue colour of sky:

According to Rayleigh scattering $\sigma_T \propto 1/\lambda^4$ i.e. the shorter the wavelength of incident radiation, stronger is the scattering by air molecules much more than the long red ones, resulting in the blue colour of the scattered light from the sky.

## 5 Red colour of sunset or sunrise:

At sunset or sunrise, the sun and its neighbouring portion appear red because sunlight travels a greater length of earth's atmosphere and reaches directly the observer's eye. The light is deprived of the blue colour due to scattering and the remaining colour appears red.

## 6 Red light used for danger signals:

Red light has longest wavelength in the visible region; therefore according to Rayleigh scattering, it is scattered the least. Consequently it can travel longer distance in atmosphere and can be seen from larger distances in comparison to other wavelengths of visible region. *That is why red light is used for danger signals even though our eye is most sensitive to yellow green.*

**Example:** *Calculate the value and dimensions of Thomson scattering cross-section.*
**Solution:** we know that Thomson scattering cross-section is given by

$$\sigma_T = \frac{8}{3}\pi r_0^2 = \frac{8}{3}\pi \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{mc^2}\right)^2 \tag{7}$$

Since $e = 1.6 \times 10^{-19}$ coulomb, $m = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^{8}$ m/s and $\frac{1}{4\pi\epsilon_0} = 9 \times 10^{9}$

$$\begin{aligned} \therefore \sigma_T &= \frac{8}{3} \times 3.14 \times \left[ 9 \times 10^{9} \times \frac{\left(1.6 \times 10^{-19}\right)^2}{9.1 \times 10^{-31} \times (3 \times 10^{8})^2} \right]^2 \\ &= \mathbf{6.65 \times 10^{-20} m^2} \end{aligned}$$

Obviously the dimensions of $\sigma_T$ are the same as that of $r_0^2$ i.e. $[L^2]$.