

Einstein A & B Coefficients and Relation Between them: B.Sc. Part-2, Hons. & Sub.

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The Einstein relations

Einstein showed that the A and B coefficients are related.

In equilibrium, the rate of change of upper and lower populations must be 0, i.e.

$$\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$$

$$\begin{aligned} \frac{dN_2}{dt} = & \text{spontaneous emission 'flow'} \\ & + \text{stimulated emission 'flow'} \\ & - \text{stimulated absorption 'flow'} \end{aligned}$$

$$\frac{dN_2}{dt} = A_2 N_2 + B_{21} N_2 - B_{12} N_1 = 0 \quad [1]$$

likewise,

$$\frac{dN_1}{dt} = -A_2 N_2 - B_{21} N_2 + B_{12} N_1 = 0 \quad [2]$$

From [1]:

$$B_{12} N_1 = A_2 N_2 + B_{21} N_2$$

Re-arrange for

$$= \frac{A_2 N_2}{B_{12} N_1 - B_{21} N_2}$$

or

$$= \frac{\frac{A_2}{B_{21}}}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \quad [3]$$

However, in thermal equilibrium, Boltzmann statistics will tell us the relative population of state 1 and state 2

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} \exp((E_2 - E_1)/kT)$$

where

g_1 = degeneracy of state 1

g_2 = degeneracy of state 2

k = Boltzmann's constant

T = temperature in Kelvin

In our case

$$h = E_2 - E_1$$

Therefore

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} \exp(-h/kT) \quad [4]$$

sub into equ. [3]

$$= \frac{\frac{A_2}{B_{21}}}{\frac{g_1 B_{12}}{g_2 B_{21}} \exp(h/kT) - 1} \quad [5]$$

Since our system is in thermal equilibrium, must be identical the black body emission, i.e.

$$= \frac{8 \pi^3 h^3}{c^3} \frac{1}{\exp(h/kT) - 1} \quad [6]$$

Equating equs. [5] and [6] we get the Einstein relations:

$$g_1 B_{12} = g_2 B_{21} \quad [7]$$

and

$$\frac{A_2}{B_{21}} = \frac{8 \pi^3 h^3}{c^3} \quad [8]$$

The ratio of the spontaneous to stimulated emission is given by:

$$\text{Ratio} = \frac{\text{spont.}}{\text{stim.}} = \frac{A_2}{B_{21}} \quad [9]$$

Re-arranging equ. [7], to get:

$$\text{Ratio} = \frac{g_1 B_{12}}{g_2 B_{21}} \exp(h \nu / kT) - 1)$$

but $g_1 B_{12} = g_2 B_{21}$, therefore:

$$\text{Ratio} = \exp(h \nu / kT) - 1$$

e.g. electric light bulb, $T = 2000\text{K}$, $\nu = 5 \times 10^{14} \text{ Hz}$

$$\text{Ratio} = 1.5 \times 10^5$$