Lecture Notes of B.Sc.(HONS.) PHYSICS ,Part-III, Paper -VI

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TOPIC:-------Macrostate and Microstate

<u>Microstate and Macrostate:</u> Let us consider an ensemble consisting of a large no. of independent systems or a gas consisting of a large no. of molecules, in the phase space. Each system or molecule may be represented by a point known as phase point or representative point in the phase space. Let the phase space is divided into cells numbered 1,2,3…..,,k etc. adjoining one another and having a volume equal to

$\delta q_1\ \delta q_2\ \delta q_3 \ldots\ldots\ldots.. \delta q_f\ \delta p_1\ \delta p_2\ \delta p_3 \ldots\ldots\ldots\ldots\ \delta p_f$.

A phase point for any system or molecule may be supposed to lie inside one of these cells. **In order to define the microstate of the ensemble we must specify the individual position of phase points for each system or molecule of the ensemble**. In other words, we must state to which cell each system or molecule belongs temporarily. For example the density is same if the no. of molecules in each volume element or ordinary space is the same regardless of which particular molecule lie in any volume element.

A macrostate of the ensemble may be defined by the specification of the no. of phase points (i.e. system or molecules) in each cell of phase space such as n_1 phase points are in cell 1,n_2 phase points are in the cell 2, n_3 phase points are in the cell 3 and so on.

Many different microstate may corresponds to the same microstate. For example let us identify the phase points as a,b,c,….etc. Let a particular microstate be specified by stating that phase points ach are in cell 1,be are in cell 2,f is in cell 3 and so on as shown in fig.(1). The corresponding macrostate is specified by giving the no. of phase points n_1=3 in cell 1,n_2=2 in cell 2,n_3= 1 in cell 3 and so on. If we interchange any two phase points from different cells say a and b, we shall have the different

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microstate, but the same macrostate. On the other hand if we interchange the two phase points in the same cell say a and c we shall have the same microstate as well as the same macrostate. If the systems of ensemble are in constant motion ,just like the molecules of gas, the ensemble is continuously and spontaneously changing from one microstate to another and almost as frequently from one macrostate to the other.

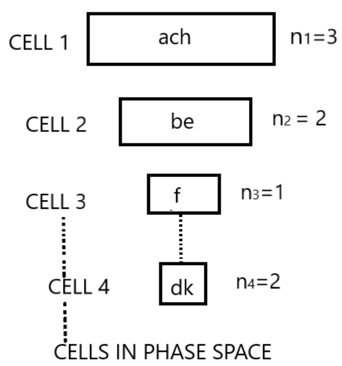

fig.(1)

The microstate which are allowed under given restriction are called **accessible microstate**. For example in the case of 3 molecule a, b, c to be distributed between two halves of a box, if none can be outside the box, then (ab,c), (a,bc), (ac,b) are accessible microstate while (a,b),(a,c),(b,c)etc. are inaccessible microstate. One of the most fundamental postulate of statistical mechanics is that all accessible microstate corresponding to possible macrostate are equally probable. This states that the probability of finding the phase point in any one region is identical with that for any other region of equal volume provided the regions corresponds equally well with the given condition . Thus this postulates is the postulate of equal a priori probability. From this postulate it follows that the probability of occurrence of a given macrostate is proportional to the no. of microstates that corresponds to that macrostate.

Thermodynamic Probability: The no. of processes by which state of a physical system can be realised. In thermodynamics a system is characterised by specific values of density, pressure, temperature etc. These values determines the state of a

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system as a whole (macrostate). However for the same density, Temp. and so on, the systems particles can be distributed in space by different process and can have different momenta. Each given distribution is called the microstate of the system. .Thermodynamic Probability is not a probability in mathematical sense. It is used to determine the properties of system which are in Thermodynamics equilibrium (for which thermodynamics probability attains a maximum value). Thermodynamic probability of a macrostate is equal to the number of microstates which realise that macrostate. Thus the probability that the ensemble will possess energy E is proportional to Ω (E) i.e. P(E) = C Ω (E) where C is constant of proportionality and Ω (E) is thermodynamic probability.

General expression for probability:

Consider a large box divided into k cells of areas $a_1, a_2, a_3 \ldots\ldots a_k$ as shown in Fig(4). Let us throw N balls into the box in a completely random manner so that no part of box is favoured. Since certain distribution occurs more than any other, we call this the most probable distribution. The most probable distribution is one in which the no. of balls in each cell is proportional to the size of the cell, a large cell has greater probability to hit than a small cell. The probability P that the ball be distributed in a certain way among the cells depends upon two factors:

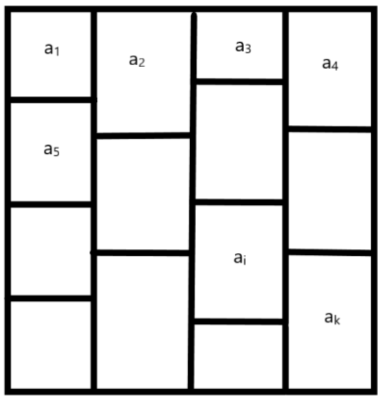

Fig.(4)

1. A priori probability (G) or the distribution which is based on properties of each cell.
2. The thermodynamic probability(Ω) of the distribution which is the no. of different accessible sequence in which the balls may be distributed among

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the cells without changing the no. in each cell. Thus the balls are explicitly assumed to identical but distinguishable.

Let g_i be the priori prob. that a ball fall in the i^{th} cell.

∴ $g_i = \frac{a_i}{A}$ where A is the area of entire box.

$(g_i)^2$ = priori prob. that two ball fall simultaneously in the i^{th} cell.

$(g_i)^{n_i}$ = priori prob. that n_i ball fall simultaneously in the i^{th} cell.

Priori prob. of distribution of N balls among K cells such that n_1 balls in 1 cell, n_2 balls in second cell etc. is given by:

G = $(g_1)^{n_1} (g_2)^{n_2} (g_3)^{n_3} \ldots\ldots\ldots\ldots (g_k)^k$

Where $N = n_1+n_2+n_3+n_4 \ldots\ldots\ldots\ldots + n_k$ = constant

If all the cells are of equal size they have same priori prob.

∴G=$(g)^{n_1+n_2+n_3 \ldots\ldots\ldots\ldots + n_k}$

∴G=$(g)^N$

Since all distribution of balls among the cells are not equally probable ,therefore we have to introduce the concept of thermodynamic probability. Thus the no. of ways in which n_1 balls out of N balls falls in 1^{st} cell is given by:

$$= \frac{N!}{n_1!\,(N-n_1)!}$$

∴ no. of ways in which n_2 balls out of remaining $(N-n_1)$ ball in second cell

$$= \frac{(N-n_1)!}{n_2!\,(N-n_1-n_2)!}$$

∴ Total no. of ways in which n_1 ball fall in cell 1, n_2 ball fall in cell 2 …….n_k in cell k are Ω=

$$\frac{N!}{n_1!\,(N-n_1)!}\ \frac{(N-n_1)!}{n_2!\,(N-n_1-n_2)!} \ldots\ldots\ldots\ldots\ldots \frac{(N-n_1-n_2 \ldots\ldots\ldots n_{k-1})}{n_k!\,(N-n_1-n_2 \ldots\ldots\ldots n_k)!}$$

$$= \frac{N!}{n_1!n_2!\ldots\ldots\ldots\ldots n_k!}$$

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$\therefore$ Total prob. =P =GΩ= $\frac{N!}{n_1!n_2!\ldots\ldots\ldots\ldots\ldots n_k!}$ $(g_1)^{n_1}$ $(g_2)^{n_2}$ $(g_3)^{n_3}$ … … … … $(g_k)^{k}$

$$=N! \prod_{i=1}^{k} \frac{(g_i)^{n_i}}{n_i!}$$

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