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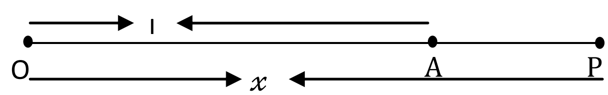
Program for B.Sc (Hons) Part-2

Topic :- Expression for work done in stretching Horizontal Elastic String.

Elastic String :- An elastic string is a string which has not fixed length i.e. it can be stretched. There are some strings which are more stretchy than others and the modulus (i.e. modulus of elasticity) of a string is a measure of how stretchy it is. The modulus is measured in newtons.

The length of an elastic string which does not have any forces acting upon it is known as the natural length of the string. If a string has been stretched, then the extension is how much longer the string is as a result of being stretched. Clearly, extension = length of the string − natural length.

Hooke's Law :- The tension of an elastic string is proportional to the extension of the string beyond its natural length.



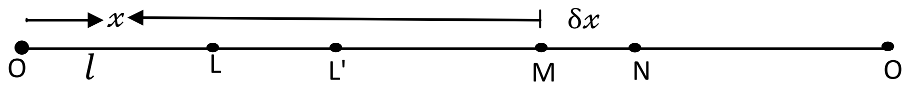
Suppose the natural length of the string OA be l and length of stretched form OP=x. Now, suppose the tension in the string be T, then according to Hooke's law,

$$T \propto \frac{\text{Extension of string}}{\text{lnatural length}}$$

$$T \propto \left(\frac{x-l}{l}\right)$$

$\Rightarrow T = \lambda.\left(\frac{x-l}{l}\right)$, where λ is the proportional Constant called modulus of elasticity of the string.

Expression for the work done against the tension in stretching an horizontal light elastic string: - The work done against the tension in stretching a light elastic string is equal to the product of its extension and the average (mean) of the initial tension and final tension.



Suppose OL be a light elastic horizontal string of natural length l (i.e. OL $=l$) whose first and 'O' is fixed. Now suppose the string be stretched firstly at the point L' and suppose OL' =b . Again, second time, the string OL' is stretched to the point O' and OO' $= \mathcal{L}'$. Now we have to find the work done against the tension is stretching string from L' to O'. Suppose λ be the modulus of elasticity of the string. Again suppose T_1 and T_2 be the tensions present in the string at the points L' and O' respectively.

Also, suppose OM=x and ON=$(x+\delta x)$.

So, MN=$(x+\delta x)$-$x = \delta x$, where δx is very small in amount with respect to x so that the tension of the string at M and N is approximately equal.

Suppose T is the tension of the string at M. If the string be stretched to a length x, then by Hooke's law, tension present at M is

$$T = \lambda . \left(\frac{x-l}{l}\right) \text{..} \quad (1)$$

and this tension acts towards O.

The initial tension L' can be written

As $$T_1 = \lambda . \left(\frac{b-l}{l}\right) \text{..} \quad (2)$$

And the final tension at O',

$$T_2 = \lambda . \left(\frac{l'-l}{l}\right) \text{..} \quad (3)$$

Now , the produced e$x$tension

= L'O' = OO' –OL' = (l'-b)

Therefore, work done against the tension in stretching the string from M to N

$$= \lambda . \left(\frac{x-l}{l}\right) .\delta x$$

So, the total work done is stretching the light elastic string form an initial position L' to O' is given by $\int_b^{l'} \frac{\lambda\,(x-l)}{l}\ .dx$

$$= \frac{\lambda}{l} \int_b^{l'} (x-l).\,dx$$

$$=\frac{\lambda}{2l}\left[(x-l)^2\right]_{b}^{l'}$$

$$=\frac{\lambda}{2l}\left[(l'-l)^2-(b-l)^2\right]$$

$$=\frac{\lambda}{2l}\,(l'-l+b-l)\,(l'-l-b+l)$$

$$=\frac{\lambda}{2l}\,(l'-b)[(l'-l)+(b-l)]$$

$$=\left(\frac{(l'-b)}{2}\right)\left[\frac{\lambda(l'-l)}{l}+\frac{\lambda(b-l)}{l}\right]$$

$$=\left(\frac{(l'-b)}{2}\right)[T_1+T_2]$$

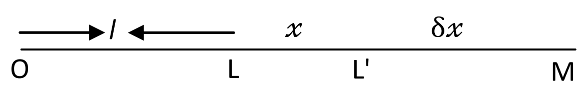
$$=(l'-b)\left[\frac{T_1+T_2}{2}\right]$$

$$=(l'-b)\left[\frac{\text{Initial tension }+\text{Final teirion}}{2}\right]$$

= (Produced extension)x (average of the initial and final tensions).

Note :- The energy of a stretched elastic string is equal to half the product of the tension and extension. That is $E=\frac{1}{2}\times$ extension $\times$ tension .

We can prove it as follows:-



Suppose l be the natural length of the string and $(l+x)$ be the stretched length. So 'x' is the extension of the string.

According to Hooke's Law, the tension can be give as $T=\lambda.\left(\frac{x}{l}\right)$

The work done in stretching the string to a distance δx is

$$T.\delta x=\lambda.\frac{x}{l}.\,\delta x$$

Again , the work done in stretching the string through a distance y beyond its natural length l is $\int_0^y \frac{\lambda}{l}\,.x.\,dx=\frac{\lambda}{l}\int_o^y x\,dx=\frac{\lambda}{l}\left[\frac{x^2}{2}\right]_0^y$

$$=\frac{\lambda}{l}.\frac{y^2}{2}=\frac{1}{2}\times y\times\left(\frac{\lambda}{l}.y\right)$$

$$=\frac{1}{2}\times\text{extension}\times\text{tension}$$

Hence the result.