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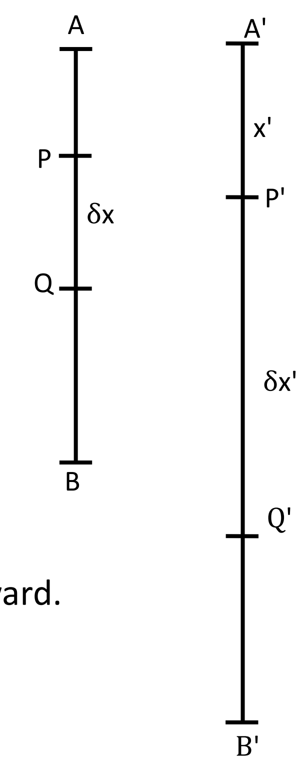
Program for B.Sc (Hons) Part-2

Topic :- Evaluation of the Extension of a Heavy Elastic string.

Statement: - To obtain the extension of a heavy elastic string of weight W and natural length l hanging from one end supporting a weight W' at the other, where λ is the modulus of elasticity of the string.

Proof: Suppose l be the natural length of heavy elastic string AB, Where A be the point of suspension

Suppose. A'B' (= L) be the length of the string when stretched. Suppose



PQ=Element of length δX when, unstretched and

P'Q'=the corresponding length the δx when unstretched.

and P'Q' = the corresponding length the δx' when stretched.

Let T= tension of the string about the depth x'

and x = length of unstretched part

x' = length of stretched part

Let W = weight of the string of length of length AB,

W' = the weight suspended from the end B' acting downward.

Therefore, $\frac{W}{l}$ is the weight of unit length of the string.

Also weight of the portion P'B'

= weight of the portion PB

$= W\left(\frac{l-x}{l}\right)$

For equilibrium of the element δx', we have tension in the element P'Q' acting upwards

= weight W' suspended from the end B' acting upward d + Weight of the portion P'B' acting downwards... (1)

But Hooke's Law says that tension T in P'Q' $=\lambda\left(\frac{\delta x'-\delta x}{\delta x}\right)$.

Therefore, equation – 1 becomes

$$\lambda\left(\frac{\delta x'-\delta x}{\delta x}\right) = W' + \frac{W}{l}\,(l - x)$$

$$\Rightarrow \quad \frac{\delta x'-\delta x}{\delta x} = \frac{W'}{\lambda} + \frac{W}{\lambda} - \frac{Wx}{l\lambda}$$

$$\Rightarrow \quad \frac{\delta x'}{\delta x} - \frac{\delta x}{\delta x} = \frac{W'}{\lambda} + \frac{W}{\lambda} - \frac{Wx}{l\lambda}$$

$$\Rightarrow \quad \frac{\delta x'}{\delta x} - 1 = \frac{W^{l}}{\lambda} + \frac{W}{\lambda} - \frac{Wx}{l\lambda}$$

$$\Rightarrow \quad \delta x' - 1 = \frac{W^{l}}{\lambda} + \frac{W}{\lambda} - \frac{Wx}{l\lambda}$$

$$\Rightarrow \quad \delta x' = \left(1 + \frac{W^{1}}{\lambda} + \frac{W}{\lambda} - \frac{Wx}{l\lambda}\right)\delta x$$

$$\Rightarrow \quad \int_0^L \delta x' = \int_o^L \left(1 + \frac{W^{1}}{\lambda} + \frac{W}{\lambda} - \frac{Wx}{l\lambda}\right)\delta x$$

$$\Rightarrow \quad [x']_o^L = \left\{1 + \frac{1}{\lambda}(W + W')\right\}[x]_o^l - \frac{W}{2l\lambda}[x^2]_o^l$$

$$\Rightarrow \quad L = \left[1 + \frac{1}{\lambda}(W + W')\right] l - \frac{Wl^2}{2\lambda l}$$

$$\Rightarrow \quad L = l + \frac{Wl}{\lambda} + \frac{W'l}{\lambda} - \frac{Wl}{2\lambda}$$

$$\Rightarrow \quad L = l + \frac{Wl}{2\lambda} + \frac{W'l}{\lambda}$$

$$\Rightarrow \quad L - l = \frac{l}{\lambda}\left[W' + \frac{W}{2}\right]$$

Hence, Extension produced

$$= \frac{l}{\lambda}\left[W' + \frac{W}{2}\right]$$

Corollaries:-

1. If the string is weight less, then W=0. So, The extension due to W' is $\frac{l}{\lambda}$ W'.
2. If the string be heavy and if no weight is hanged, i.e. hanging under its own weight only, then W'=o. So, the extension produced $= \frac{lw}{2\lambda}$.