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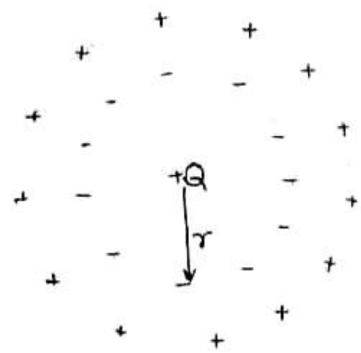
Debye Length

Consider a charge +Q placed in plasma. It will attract electrons all around and the ions will be repelled by this. In equilibrium, the charge produces electric field around it then



$\vec{E} = -\nabla \emptyset$ (i.e. gradient of scalar potential $\emptyset$) ……………… (i)

At a distance r from test charge Q there is a charge particle, then potential energy

$P.E = -e\,\emptyset$ for electrons

$P.E = e\,\emptyset$ for ions

Boltzmann's law says that electrons will have a tendency to go in a region of small potential energy, then

Electron density $n_e = n_0\, e\, \frac{e\emptyset}{KT_e}$

Ion density $n_i = n_0\, e\, \frac{-e\emptyset}{KT_i}$

Where n_o = equilibrium density of plasma electrons which is same as plasma ions

For simplicity $T_e = T_i = T$

Poisson's equation in electrostatics

$$\varepsilon_0 \nabla \vec{E} = \rho$$

Where ρ=charge density

$\rho = n_i\, e - n_e\, e$

$\rho = (n_i - n_e)e$

$$\because \nabla \vec{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \vec{E} = \frac{e}{\varepsilon_o} (n_i - n_e)$$

From eqn. (i) $\nabla.(-\nabla\emptyset) = \frac{e}{\varepsilon_o}(n_i - n_e)$

$$\nabla^2\emptyset = \frac{e}{\varepsilon_o}(n_e - n_i)$$

If $\frac{e\emptyset}{KT} \ll 1$, then

$$n_e \simeq n_o\left(1+\frac{e\emptyset}{KT}\right)$$

$$n_i \simeq n_o\left(1-\frac{e\emptyset}{KT}\right)$$

$$\therefore \quad \nabla^2\emptyset = \frac{en_0}{\varepsilon_o}\left[1+\frac{e\emptyset}{KT} - 1 + \frac{e\emptyset}{KT}\right]$$

$$\nabla^2\emptyset = \frac{2n_o e^2}{\varepsilon_o KT}\emptyset$$

The potential due to a charge q as a function of distance. So in that case

$$\frac{1}{r^2}\frac{\delta}{\delta r}\left(r^2\frac{\delta\emptyset}{\delta r}\right) = \frac{2n_o e^2}{\varepsilon_o KT}\emptyset$$

The coefficient in the right hand side that multiplies by the quantity $\emptyset$ has one upon length square then

$$\lambda^2_D = \frac{\varepsilon_0 KT}{2n_0 e^2}$$

λ_D has a dimension of length. It depends upon temperature and electron density.

Poisson's equation can be written as $\frac{I}{r^2}\frac{\delta}{\delta r}\left(r^2\frac{\delta\emptyset}{\delta r}\right) = \frac{\emptyset}{\lambda^2_D}$

Now solve this equation

By the fact that the potential of charge placed in free space goes as

$\emptyset = \frac{I}{r}F(r)$ Where F (r) is some constant

Substitute in above equation

$$\frac{d^2F}{dr^2} = \frac{F}{\lambda^2_D}$$

The solution of above equation is simple exponential solution

$$F = C_1 e^{\frac{-r}{\lambda_D}}$$

It has a physical argument that the potential must decrease as more away from charge.

Now put F in $\emptyset$

$$\emptyset = \frac{C_1}{r} \quad e^{\frac{-r}{\lambda_D}}$$

To evaluate C_1 the screening effect of electrons or ions will not effective because in an electron cloud or ion cloud, the electric field is zero

If $r << \lambda_D$ then potential

$$\emptyset = \frac{Q}{4\pi\varepsilon_0 r} \quad , \quad C_1 = \frac{Q}{4\pi\varepsilon_0}$$

Potential can be written as

$$\emptyset = \frac{Q}{4\pi\varepsilon_0} e^{\frac{-r}{\lambda_D}}$$

The term $e^{\frac{-r}{\lambda_D}}$ is extra screening factor and the potential will full off quite rapidly when $r > \lambda_D$

So λ_D = Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 KT}{2n_o e^2}}$$

The significance is that the electrostatic potential of a charge is felt over a distance of the order of λ_D beyound which this is screen out.

So dimension of plasma should be bigger than Debye length.