Lecture Notes of B.Sc.(HONS.) PHYSICS ,Part-II, Paper -IV

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TOPIC:-------Radioactive Disintegration

What is Radioactive Disintegration?: When an unstable radioactive element constantly breaks up into new radioactive atoms with the emission of either α-particles or β- particles and some times with the γ-rays. The new atoms in general radioactive and have entire new chemical and radioactive properties. The spontaneous breaking up of a nucleus is known as radioactive disintegration. The atoms of radioactive substance undergo continuous decay. Hence a series of a new radioactive elements is produced by disintegration until a stable element is obtained. When a radioactive element disintegrates by emission of α rays it is turned into new element of atomic number two less and atomic weight four less than of the parent element. Similarly, when an element disintegrates by the emission of β-rays, it is tuned into another new element of atomic number one greater than that of the parent element but of the same atomic weight. The disintegration of Uranium into Thorium by emission of alpha particle and then to protactinium by the emission of beta particle is expressed as follows:

$$_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He$$

(Uranium) (Thorium) (α-particle)

$$_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}\beta^{0}$$

(Thorium) (Protactinium) (beta Particle)

This process of disintegration continues till the stable lead nucleus forms at the end.

Rules of Radioactive Displacement :

1. When a radioactive atom emits an α-particle it is converted into new atom, the product atom shifts in the periodic table two steps in the direction of lower atomic number and its mass number is lowered by 4 units.
2. When a radioactive atom emits a β-particle, the product atom shifts in the direction of increasing atomic number. Both of these are called rules of radioactive displacement.

Rutherford-Soddy Theory of Radioactive Disintegration :

Rutherford and Soddy studied the radioactive decay and formulated a theory which is based on the following laws:

1. The radioactive emission is characteristic of the isotope, it varies from one isotope to another of the same element.
2. The emission occurs spontaneously and cannot be speeded up or slowed down by physical means such as change of pressure or temperature.
3. The disintegration occurs at random and which atom will disintegrate first is simply a matter of chance.
4. The rate of disintegration of a particular substance (i.e., number of atoms disintegrating per second) at any instant is proportional to the number of atoms present at that instant.

Let N be the number of atoms present in a radioactive substance at any instance t. Let dN be the number that disintegrates in a short interval *dt*. *Then* the rate of disintegration is - *dN/dt*, *a*nd is proportional to N i.e.,

$$-\frac{dN}{dt} = \lambda N \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where λ is a constant for the given substance is called 'Decay Constant' or

'Disintegration Constant' or 'Radioactive Constant' or 'Transformation Constant'.

Therefore $\frac{dN}{N} = -\lambda\, dt$

Integrating we get,

$log_e N = -\lambda t + C$ ……………………………………………………..(2)

Where C is the integration constant. To determine C we apply the initial condition.

Suppose there were N_0 atoms in the beginning i.e. $N = N_0$ at $t=0$, then

$log_e N_0 = C$

$log_e N = -\lambda t + log_e N_0$

$log_e \frac{N}{N_0} = -\lambda t$

$\frac{N}{N_0} = e^{-\lambda t}$

$$\mathbf{N = N_0\, e^{-\lambda t}}$$ ………………………………………………………………..(3)

This equation shows that the number of atoms of a given radioactive substance decreases exponentially with time *(i.e.*, more rapidly at first and slowly afterwards). as shown in Fig.(1) This is the **Rutherford-Soddy Law of radioactive decay**.

Instead of plotting N (or activity) against t, we rather plot log, *N*[or log $_e$] against t, according to relation $log_e N = \log No - \lambda t$ and get straight line Fig.(2), because the eye can judge whether or not a curve is represented by an exponential function. Moreover the logarithmic scale readily represents a range much wider than that represented by a linear scale.

It the product nucleus p is stable and not radioactive, the number of these nuclei present at time t is given by

$N_p = N_0 - N = N_0 (1 - e^{-\lambda t})$ …………………………………..(4)

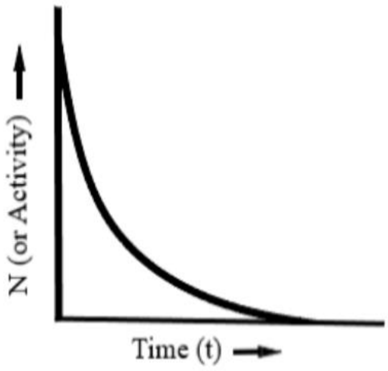

Fig.(1)

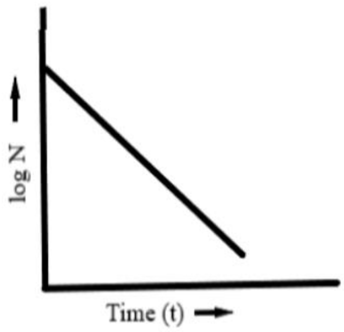

Fig.(2)

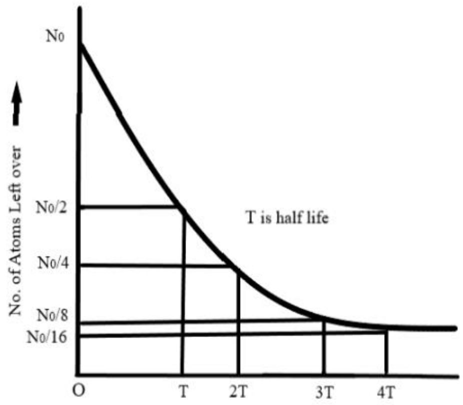

Fig.(3)

eqn.(3) and (4) constitute the law of radioactive decay and are obeyed no matter how the external condition, such as temperature, pressure are verified. The radioactive decay is independent of the chemical composition of the substance containing the nuclei and this confirms that the **decay process is indeed a nuclear process**.

Substituting t=1/ λ in eqn. (3) ,we get

$N=N_0 \exp(-\lambda * 1/\lambda) = N_0 e^{-1}$

$N=\frac{1}{e} N_0$

Therefore the **radioactive constant is defined as the reciprocal of the time in which number of atoms of a radioactive substance falls to 1/e times its initial value.**

<u>Half-Life:</u> The time interval T in which the mass of a radioactive substance, or the number of its atoms, is reduced to half of its initial value, is called the "**half-life**" of that substance. The half-life of a radioactive substance is constant, but it is different for different substances. It can be read from the graph in Fig.(3)

<u>Relation between Half-Life and Decay constant:</u>

Let N_0 be the number atoms present in a radioactive substance at time t=0, and N the number at a later time. Then by Rutherford =Soddy Law, we have

$N = N_0 e^{-\lambda t}$

Now at time t=T (half-life period), N=N_0/2.

$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T}$

$\Rightarrow \frac{1}{2} = e^{-\lambda T}$

$\Rightarrow e^{\lambda T} = 2$

$\Rightarrow \lambda T = \log_e 2$

$\Rightarrow T = \log_e 2 / \lambda = \frac{\log_e 10 \log_{10} 2}{\lambda}$

$\Rightarrow T = \frac{2.3026*0.3010}{\lambda}$

$\therefore \boxed{T = \frac{0.693}{\lambda}}$

This is the **relation between Half-life and Decay constant**. The half-life of a radioactive substance cannot be changed by any physical or chemical change. The half-life of an isotope of lead ($_{82}Pb^{214}$) is 26.8 minutes. If the isotope forms some compound by chemical combination ,even then its half-life will be 26.8 minutes.

Average Life (or Mean-Life) of Radioactive Atom: The average life of a radioactive atom is equal to the sum of the life times of all the atoms divided by the total number of atoms.

Relation between Mean-Life time and Decay Constant:

Suppose N_0 is the total no. of atoms at time t=0, and N is the no. remaining at instant. Suppose a no. dN of them disintegrates between t and t+dt. As the interval dt is small we may assume that each pf these dN atoms had a life time of t seconds. Thus the total life time of dN atoms is t dN.

Since the disintegration process is a statistical one ,any single atom may have a life from 0 to infinity. Hence the sum of life times of all the N_0 atoms is given by;

$$\int_{t=0}^{t=\infty} t dN$$

Dividing it by N_0 ,total no. of atoms originally present,we get the average life time $\bar{T}$ of an atom. Therefore

$$\bar{T} = \frac{\int_{t=0}^{t=\infty} t dN}{N_0}$$

Now from the disintegration law, we have

$N = N_0 e^{-\lambda t}$ or $dN = -N_0 \lambda e^{-\lambda t} dt$

Substituting in the above expression we get,

$$\bar{T} = \frac{\int_{t=0}^{t=\infty} t(N0 \ \lambda e^{-\lambda t} dt)}{N_0} = \lambda \int_{t=0}^{t=\infty} t e^{-\lambda t} dt$$

Integrating by parts, we get,

$$\bar{T} = \lambda \left[\frac{te^{-\lambda t}}{-\lambda}\right]_0^{\infty} - \lambda \int_0^{\infty} \frac{e^{-\lambda t}}{-\lambda} dt = \left[-te^{-\lambda t}\right]_0^{\infty} + \int_0^{\infty} e^{-\lambda t} dt$$

The first term ,on substituting the limits becomes zero, Therefore

$$\bar{T} = \int_0^{\infty} e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^{\infty} = \left[0 - \frac{1}{-\lambda}\right] = \frac{1}{\lambda}$$

Thus the mean life of a radioactive atom is equal to the reciprocal of its disintegration constant.

Relation between Mean-life and Half-life:

$\bar{T} = \frac{1}{\lambda}$ and $T = \frac{0.693}{\lambda}$

$$\boxed{\bar{T} = \frac{T}{0.693} = 1.44T}$$

Note : Thus mean life is longer than half-life