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Title of Topic – Transport Phenomena

Name of Program –Physics (Hons), Part –I, Paper –II,Group A Heat

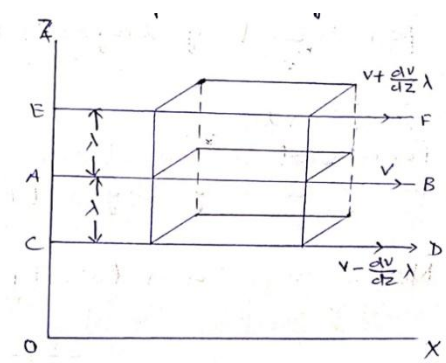
Transport phenomena

According to kinetic theory of gases, the molecules of a gas in a state of thermal agitation Therefore gas attains equilibrium state by transporting momentum, heat (thermal energy) and mass form one layer of gas to another layer giving rise to the viscosity, conductivity and diffusion respectively. This phenomena is called transport phenomena.

The transport phenomena occur only in non-equilibrium state of a gas.

Viscosity: Transport of momentum

There is a relative motion of different layers of a non-equilibrium gas with respect to one another. The layer moving faster will impart momentum to the layer moving slower to bring about an equilibrium state. Thus the transport of momentum gives rise to phenomenon of viscosity.



Consider a layer of gas AB moving with velocity V at a distance Z form O.

Velocity gradient =$\frac{\mathrm{dv}}{\mathrm{dz}}$

Consider two layers EF & CD just above and below AB respectively at a distance λ equal to mean free path of molecule.

Velocity of gas in the layer EF = $V+\frac{\mathrm{dv}}{\mathrm{dz}}\lambda$

Velocity of gas in layer CD= $V-\frac{\mathrm{dv}}{\mathrm{dz}}\lambda$

Let n =Number of molecule per unit volume

m = mass of each gas molecule

Due to thermal agitation the gas molecules are moving in all possible directions. $ie\,\frac{1}{3}$ rd molecules moving in each three directions parallel to three co-ordinate axis.

So that average $\frac{1}{6}$ th of molecules more parallel to any one axis in one particular direction *ie +ve* or *– ve* direction.

Number of molecule passing downward from EF to CD per unit area per sec. = $\frac{\mathrm{nc}}{6}$

Forward momentum lost per unit area per second by layer EF = $\frac{\mathrm{mnc}}{6}\left(V+\frac{\mathrm{dv}}{\mathrm{dz}}\lambda\right)$.

Similarly

Forward momentum gained per unit area per sec. by layer EF = $\frac{mnc}{6}\left(V - \frac{dv}{dz}\lambda\right)$.

Net momentum lost by the layer EF per unit area per second

$$= \frac{mnc}{6}\left\{\left(V + \frac{dv}{dz}\lambda\right) - \left(V - \frac{dv}{dz}\lambda\right)\right\}.$$

$$= \frac{1}{3} mnc\,\lambda\,\frac{dv}{dz}.$$

Backward dragging force per unit area = Gain or loss in momentum per unit area of layer AB

$$F = \frac{1}{3} mnc\,\lambda\,\frac{dv}{dz}$$

This must be equal to the viscous force or tangential force $\left(\eta \frac{dv}{dz}\right)$ per unit area

η= coefficient of viscosity

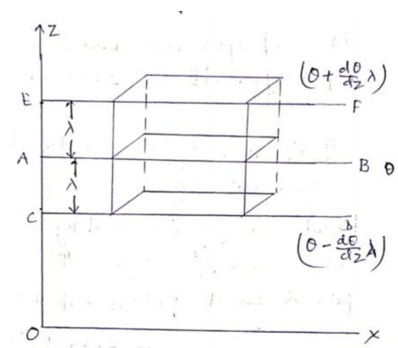
$$\eta\frac{dv}{dz} = \frac{1}{3} mnc\,\lambda\frac{dv}{dz}$$

$$\eta = \frac{1}{3} mnc\,\lambda, \quad \eta = \frac{1}{3}\rho c\,\lambda$$

where $\rho = mn$ = Density of gas.

Thermal Conductivity : Transport of thermal Energy

The different layers of gas may have different temperatures. The molecules will carry Kinetic energy (heat) from the regions of higher temperature to the region of lower temperature to bring on equilibrium state. Thus the transport of thermal energy gives rise to the phenomenon of thermal conductivity.



Let θ be the temperature of layer of gas AB

Temperature gradient $= \frac{d\theta}{dz}$ along oz

Temperature of layer EF at a distance λ above AB is $= \left(\theta + \frac{d\theta}{dz}\lambda\right)$

Temperature of layer CD at a distance λ below AB is $= \left(\theta - \frac{d\theta}{dz}\lambda\right)$

This means that the gas above the layer AB is at higher temperature than the gas below AB. Hence the molecules of gas coming from EF and passing downward across AB possesses more kinetic energy than the molecules coming from CD and passing upward across AB till the equilibrium is reached.

Let n = Number of molecules per unit volume

m = mass of each molecule

c = Average velocity of gas molecule

Number of molecules crossing per unit area of layer AB upward or downward per second $\frac{nc}{6}$

Mass of gas molecule crossing per unit area of layer AB upward or downward per second $= \frac{\mathrm{mnc}}{6}$

If C_v is the specific heat of gas at constant volume then

Heat energy carried by molecules in crossing unit area of layer AB in downward direction per second = Mass x Sp. Heat x temperature

$$= \frac{\mathrm{mnc}}{6} C_v (\theta + \frac{d\theta}{dz} \lambda).$$

Similarly

Heat energy carried by molecules in crossing in unit area of layer AB in upward direction per second $= \frac{\mathrm{mnc}}{6} C_v (\theta - \frac{d\theta}{dz} \lambda)$.

Net transfer of heat energy per unit area of layer AB in downward direction per sec.

$$Q = \frac{\mathrm{mnc}}{6} C_v \left\{ \left(\theta + \frac{d\theta}{dz}\lambda\right) - \left(\theta - \frac{d\theta}{dz}\lambda\right) \right\}.$$

$$Q = \frac{\mathrm{mnc}}{6} C_v . 2\lambda \frac{d\theta}{dz}$$

$$Q = \frac{1}{3} \mathrm{mnC}\lambda \ C_v \frac{d\theta}{dz}$$

$$Q = \frac{1}{3} \rho c \lambda \ C_v \frac{d\theta}{dz}$$

Where mn $=\rho$ =density of gas

The coefficient of thermal conductivity (K) of the gas is defined as the quantity of heat flows per unit area per second to maintain unit temperature gradient

$$ie \qquad Q = K \frac{d\theta}{dz}$$

$$\therefore K \frac{d\theta}{dz} = \frac{1}{3} \rho c \lambda C_v \frac{d\theta}{dz}$$

$$K = \frac{1}{3} \rho c \lambda C_v$$

<u>Effect of temperature of K :-</u>

Coefficient of thermal conductivity $K = \frac{1}{3} \mathrm{mnC_v}\lambda$

But Mean free path $\qquad \lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$

$$K = \frac{1}{3} \mathrm{mnc} \ C_v \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

$$K = \frac{\mathrm{mc} \ C_v}{3\sqrt{2}\pi\sigma^2}$$

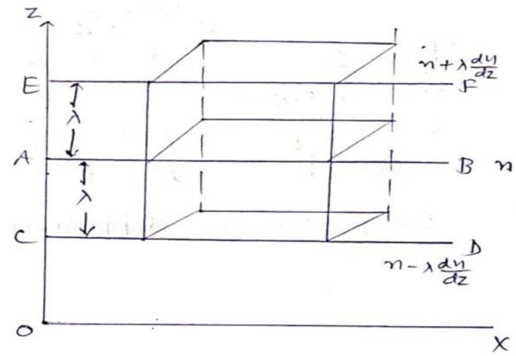
But Average molecular speed, $C \propto \sqrt{T}$

$$\therefore K \propto \sqrt{T}$$

Thus the coefficient of thermal conductivity is directly proportional to square root of absolute temperature.

Diffusion : Transport of mass

The phenomenon of diffusion is due to the transport of mass from a region of higher concentration to a region of lower concentration to bring about an equilibrium.



Let n be the concentration of layer of gas AB.

Concentration gradient = $\frac{dn}{dz}$ along oz

Consider two layer EF & CD just above and below AB respectively at a distance λ equal to mean free path

Concentration of layer EF $= n+\lambda\frac{dn}{dz}$

Concentration of layer CD $= n-\lambda\frac{dn}{dz}$

Number of molecules coming from layer EF and crossing AB in downward per unit area per second $=\frac{1}{6}c(n+\lambda\frac{dn}{dz})$

Number of molecules coming from layer EF and crossing AB in upward per unit area per second $=\frac{1}{6}c(n-\lambda\frac{dn}{dz})$

Net number of molecules crossing per unit area per second of layer AB in downward direction $=\frac{1}{6}c\left\{\left(n+\lambda\frac{dn}{dz}\right)-(n-\lambda\frac{dn}{dz})\right\}$

$$=\frac{1}{3}c\lambda\frac{dn}{dz}$$

The coefficient of diffusion is defined as the ratio of the number of molecules crossing per unit area per sec. to the rate of change of concentration with distance

Coefficient of diffusion $D = \dfrac{\frac{1}{3}c\lambda\frac{dn}{dz}}{\frac{dn}{dz}}$

$$D = \frac{1}{3}c\lambda$$

Effect of temperature and pressure

Mean free path $\lambda = \frac{KT}{\sqrt{2}\pi\sigma^2 p}$ & $C = \sqrt{\frac{8KT}{\pi m}}$

$$\therefore D = \frac{1}{3}\frac{KT}{\sqrt{2}\pi\sigma^2 p}\sqrt{\frac{8KT}{\pi m}}$$

$$D = \frac{2}{3}\frac{1}{\sigma^2 p\sqrt{m}}\left(\frac{KT}{\pi}\right)^{3/2}$$

Thus the coefficient of diffusion is directly proportional to $T^{3/2}$ and inversely proportional to pressure P.