Special Theory of **Relativity**, Michelson Morley Experiment, Physics Hons. Part1, Paper-1

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Michelson-Morley experiment

Let us consider one-dimensional wave equation of sound wave,

$$\frac{\partial^2\phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}\ . \tag{1}$$

The fluid in which the sound propagates is assumed to be at rest. But suppose the fluid is not at rest. We may investigate this by shifting to a moving frame, defining $x' = x - ut$, with $y' = y$, $z' = z$ and of course $t' = t$. This is a Galilean transformation. In terms of these new variables,

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'} \qquad , \qquad \frac{\partial}{\partial t} = -u\,\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\ . \tag{2}$$

The wave equation is then

$$\left(1 - \frac{u^2}{c^2}\right)\frac{\partial^2\phi}{\partial x'^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t'^2} - \frac{2u}{c^2}\frac{\partial^2\phi}{\partial x'\,\partial t'}\ . \tag{3}$$

Clearly the wave equation acquires a different form when expressed in the new variables (x', t'), *i.e.* in a frame in which the fluid is not at rest. The general solution is then of the modified d'Alembert form,

$$\phi(x', t') = f(x' - c_{\mathrm{R}}t') + g(x' + c_{\mathrm{L}}t') \ , \tag{4}$$

where $c_{\mathrm{R}} = c - u$ and $c_{\mathrm{L}} = c + u$ are the speeds of rightward and leftward propagating disturbances, respectively. Thus, there is a *preferred frame of reference* – the frame in which the fluid is at rest. In the rest frame of the fluid, sound waves travel with velocity c in either direction.

Light, as we know, is a wave phenomenon in classical physics. The propagation of light is described by Maxwell's equations,

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi\rho \qquad \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{5}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \qquad \boldsymbol{\nabla} \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \ , \tag{6}$$

where ρ and $\boldsymbol{j}$ are the local charge and current density, respectively. Taking the curl of Faraday's law, and restricting to free space where $\rho = \boldsymbol{j} = 0$, we once again have (using a Cartesian system for the fields) the wave equation,

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \ . \tag{7}$$

Preveiously it was assumed as the theory of sound, that there was in fact a preferred reference frame for electromagnetic radiation – one in which the medium which was excited during the EM wave propagation was at rest. This medium was called the *ether*. Also, it was generally assumed during the 19$^{\text{th}}$ century that light, electricity, magnetism, and heat all had separate ethers. It was Maxwell who realized that light, electricity, and magnetism were all unified phenomena, and accordingly he proposed a single ether for electromagnetism. It was believed at the time that the earth's motion through the ether would result in a drag on the earth.

In 1887, Michelson and Morley set out to measure the changes in the speed of light on earth due to the earth's movement through the ether (which was generally assumed to be at rest in the frame of the Sun). The Michelson interferometer is shown in figure. Suppose the apparatus is moving with velocity $u\,\hat{\boldsymbol{x}}$ through the ether. Then the time it takes a light ray to travel from the half-silvered mirror to the mirror on the right and back again is

$$t_x = \frac{\ell}{c+u} + \frac{\ell}{c-u} = \frac{2\ell c}{c^2 - u^2} \ . \tag{8}$$

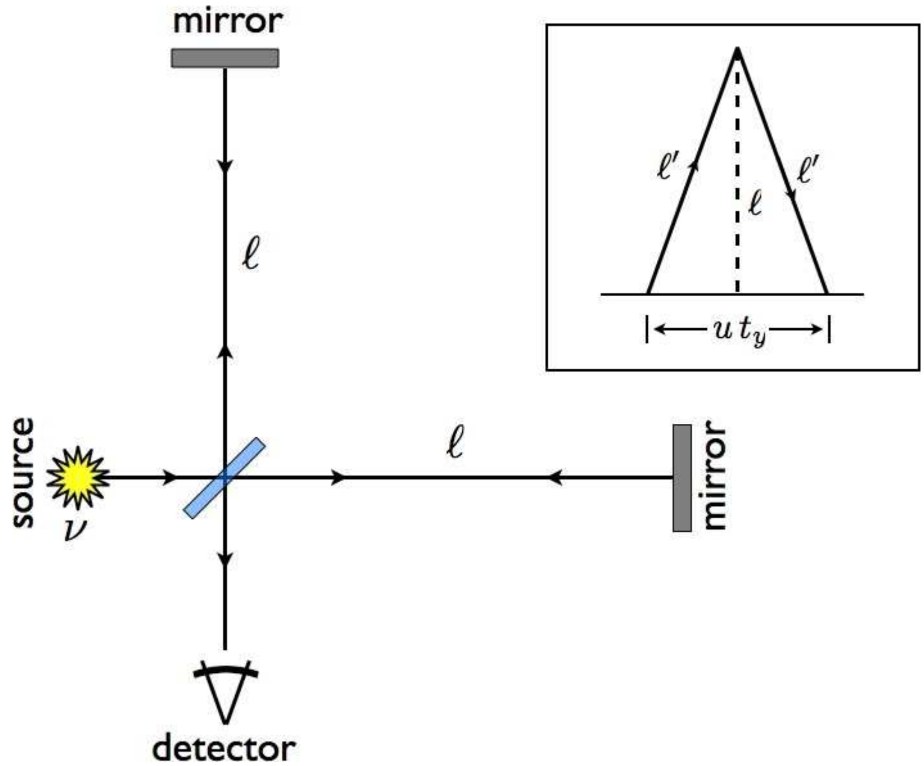

Figure 1: The Michelson-Morley experiment (1887) used an interferometer to effectively measure the time difference for light to travel along two different paths.

For motion along the other arm of the interferometer, the geometry in the inset of figure shows $\ell' = \sqrt{\ell^2 + \frac{1}{4}u^2t_y^2}$, hence

$$t_y = \frac{2\ell'}{c} = \frac{2}{c}\sqrt{\ell^2 + \tfrac{1}{4}u^2t_y^2} \qquad \Rightarrow \qquad t_y = \frac{2\ell}{\sqrt{c^2 - u^2}} \ . \tag{9}$$

Thus, the difference in times along these two paths is

$$\Delta t = t_x - t_y = \frac{2\ell c}{c^2} - \frac{2\ell}{\sqrt{c^2 - u^2}} \approx \frac{\ell}{c} \cdot \frac{u^2}{c^2} \ . \tag{10}$$

Thus, the difference in phase between the two paths is

$$\frac{\Delta\phi}{2\pi} = \nu\,\Delta t \approx \frac{\ell}{\lambda} \cdot \frac{u^2}{c^2} \ , \tag{11}$$

where λ is the wavelength of the light. We take $u \approx 30\,\mathrm{km/s}$, which is the earth's orbital velocity, and $\lambda \approx 5000\,\text{Å}$. From this we find that $\Delta\phi \approx 0.02 \times 2\pi$ if $\ell = 1\,\mathrm{m}$. Michelson and Morley found that the observed fringe shift $\Delta\phi/2\pi$ was approximately 0.02 times the expected value. The final conclusion was that the speed of light did not depend on

the motion of the source.