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Programme-05 for B.Sc. (Hon.) Part-2

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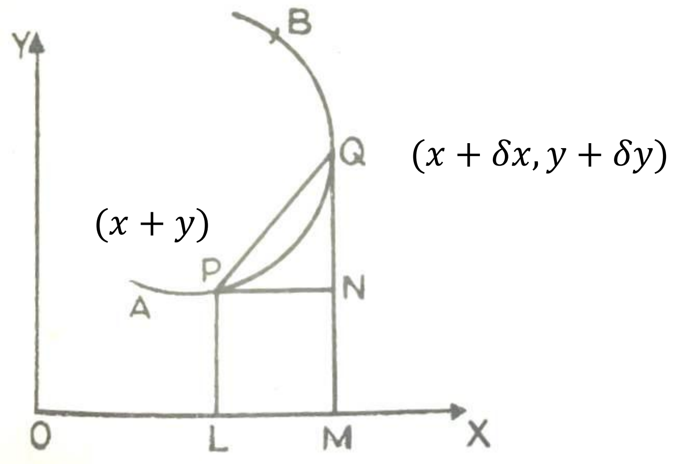
MOTION IN TWO DIMENSIONS

If a particle moves in a plane in both the direction say x-axis and y-axis, then its motion is called motion in two dimensions.

(Expression for velocity of a particle in plane curvilinear motion in terms of its rectangular Cartesian co-ordinates):-



Let there be a particle moving along the curve AB in a plane. Let O be the origin and OX & OY be the two Cartesian rectangular axes. Also let the particle is present at any time t at P and after the line Δt it reaches at Q. The co-ordinates of P and Q are taken as $P(x, y)$ and $Q(x+\delta x, y+\delta y)$ respectively.

Clearly PQ is the displacement of the particle in the time interval Δt. Let $PL \perp OX, QM \perp OX$ and $PN \perp QM$.

So, displacement of the particle parallel to OX in the time $\delta t = PN = \delta x.$

Therefore the average rate of displacement parallel to

$$X - axis = \frac{\delta x}{\delta t}$$

The rate of displacement parallel to

$$X - axis = \lim_{\delta t \to o} \frac{\delta x}{\delta t} = \frac{dx}{dt}$$

That is, the velocity at $P(x, y)$ parallel to the $X - axis$ denoted by

$$\dot{x} = \frac{dx}{dt} \qquad \text{-------------->} \ (1)$$

Similarly, the velocity at $P(x, y)$ parallel to the $y - axis$ denoted by

$$\dot{y} = \frac{dy}{dt} \qquad \text{-------------->} \ (2)$$

Let V be the magnitude of the velocity at time t. If the velocity V makes an angle θ with the positive direction of X-axis, then

$$\text{V} = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} \qquad \text{-------------->} \ (3)$$

$$\dot{y} = \frac{dy}{dt} \qquad \text{-------------->} \ (4)$$

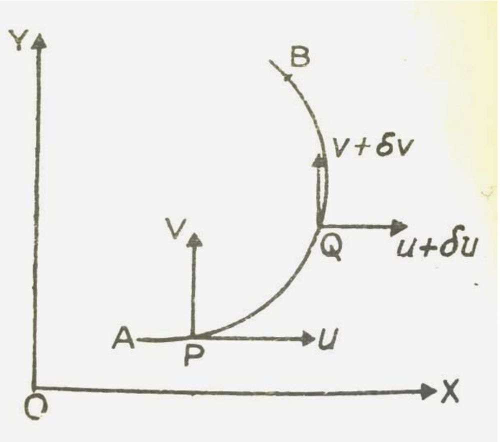
Equation – (3) is the required expression for velocity at the point $P(x, y)$ and equation – (4) gives the direction at P.

Note:- (Expression for acceleration):-

Here $u = \frac{dx}{dt}, v = \frac{dy}{dt}$

It is clear that the acceleration at p parallel to x-axis & y-axis respectively are

$$\ddot{x} = \frac{d^2x}{dt^2}, \ddot{y} = \frac{d^2y}{dt^2}$$



f=magnitude of acceleration at P

$$= \sqrt{(\frac{d^2x}{dt^2})^2 + (\frac{d^2y}{dt^2})^2}$$

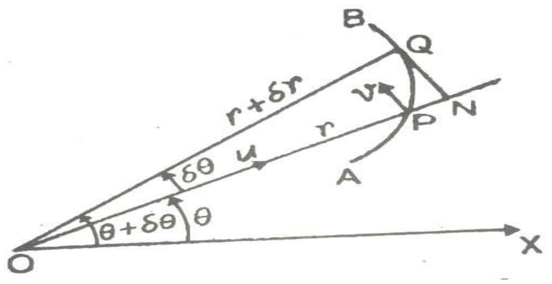
If the acceleration makes an angle θ with the positive direction of x-axis. Then

$$\tan\theta = \frac{d^2y/d^2t}{d^2x/dt^2} = \frac{\ddot{y}}{\ddot{x}}$$

(Expression for radial velocity and transverse velocity):-

Radial velocity:- If a particle moves along a curve with reference to the polar co-ordinates, then the velocity compound at a point along the radius vector is called radical velocity.

Transverse velocity:- If a particle moves along a curve with reference to the polar co-ordinates, then the velocity component at a point normal to the radius vector, is called transverse velocity.



Let O be the origin and OX the initial line. Again let a particle is moving along the curve AB and at any time t, the position of the particle is P(r, θ). Also, the position of the particle at time (t+Δ t) is Q $(r+\delta r, \theta+\delta\theta)$

Now, let QN$\perp OP$ then PN is the displacement of the particle measured along OP in the time ST and NQ is the displacement of the particle measured perpendicular to OP in the same time. Let U and V the respective velocity of the particle at P along and perpendicular to OP.

Therefore the component of velocity of P along OP is

$$U = \lim_{\delta t \to o} \frac{PN}{\delta t}$$

$$= \lim_{\delta t \to o} \frac{ON - OP}{\delta t}$$

$$= \lim_{\delta t \to o} \frac{(r+\delta r)\cos\theta - r}{\delta t}$$

But from trigonometry, we known that

$$\cos\theta = 1 - \frac{(\delta\theta)^2}{2!} + \frac{(\delta\theta)^4}{4!} \ldots\ldots\ldots\ldots..$$

At $\delta \mathrm{t} \to 0$, we shall get $\delta\theta$ smaller. Therefore, $\cos\delta\theta = 1$

(by neglecting higher power of small quantities).

Therefore, from the equation-(1), we get

$$U = \lim_{\delta t \to o} \frac{(r+\delta r)\cdot 1 - r}{\delta t}$$

$$= \lim_{\delta t \to o} \frac{r+\delta r - r}{\delta t}$$

$$= \lim_{\delta t \to o} \frac{\delta r}{\delta t} = \frac{dr}{dt} = \dot{r}$$

Therefore, the radial velocity $= \dot{r} = \dfrac{dr}{dt}$ --------------> (2)

Equation (2) is the expression for radial velocity. Similarly, the component of velocity normal to OP is given by

$$V = \lim_{\delta t \to o} \frac{QN}{\delta t}$$

$$= \lim_{\delta t \to o} \frac{(r+\delta r)\,\mathrm{Sin}\,\delta\theta}{\delta t}$$

$$= \lim_{\delta t \to o} (r+\delta r).\left(\frac{\mathrm{Sin}\,\delta\theta}{\delta\theta}\right)\left(\frac{\delta\theta}{\delta t}\right)$$

AS $\because \delta r \to O, \delta\theta \to O$ as $\delta t \to O,$ we have

$$\mathrm{V} = \mathrm{r}.1.\frac{d\theta}{dt}$$

$$= r.\frac{d\theta}{dt}$$ --------------> (3)

Therefore, transverse velocity, $\mathrm{V} = \mathrm{r}.\dfrac{d\theta}{dt}$

This is the expression for transverse velocity.

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RELATIONS

Introduction:- The word 'relation' is very common to our daily life. Relation connects two different individuals or two different things. There exist so many relations around us. For example father, mother, sister, brother, teacher, cousion, friend etc.

In mathematics relation is also equally important. It corrects two different entities. By the help of relation we become sufficient to observe the nature of one thing relative to another thing. For ex:- equality, inequality, similarity, congruency etc.

(Definition of relation):- Let A and B be the two sets, then a subset of either (AxB) or (BxA) is called a relation from A to B or B to A respectively.

Ex:- (1) A = {1,2,3}, B={4,5}

(AxB) = {(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)}

R_1 = {(1,4), (1,5)}

R_2 = {(2,5), (3,4), (3,5)}

(AxB) = universal relation.

(Domain and Range of a relation):-

Let A and B be the two sets. Now, we define a relation from A to B as

$$R = \{(x, y) : x \in A, y \in B\}$$

Here, domain of R = domain R = $\{x : x \in A\}$ for $(x.y) \in R$

Range of

$$R = \{y : y \in B for (x.y) \in R\}$$

$$\mathrm{Ex:-A} = \{a, b\}, \mathrm{B} = \{c, d, e\}$$

$$\mathrm{R} = \{(a, c), (a, e), (b, e)\}$$

$$\mathrm{dom.R} = \{a, b\}, \mathrm{range} = \{c, e\}.$$

(Relation on a set):- Let A be a set. Now, any subset of (A $\times$ A) is called a relation on A. Moreover, a subset $R \subseteq (A \times A)$ is called a relation on A.

Ex:- A={$a_1.a_2.a_3$}

R={($a_1.a_2$), ($a_1.a_3$)

(Different types of relation on a set) :-

1. **<u>Reflexive relation</u>: -** Let A be a set. Now a relation R on A is called reflexive if a R a V a ε A. That is R={(a, a): for all aεA}

   Ex:- A= {1,2,3}

   R_1 = {(1,1), (2,2), (3,3)} reflexive

   R_2 = {(1,1), (2,2), (3,3), (1,2), (2,1)} reflexive

$R_3 = \{(1,1), (2,2), (1,2), (2,1)\}$

Is not reflexive since $(3,3) \notin R_3$

2. **Symmetric Relation** : - A relation R on A is called symmetric if $aRb \Rightarrow bRa \forall a,b \in A.$

   That is if $(a,b) \in R \Rightarrow (b,a) \in R \ \forall a,b \in A.$

   Ex: 1. A={1,2,3}

   R = {(1,1), (2,2), (3,3), (1,2), (2,1)}

3. **Anti Symmetric relation**:- A relation R on A is called anti Symmetric if aRb and bRc $\Rightarrow a = b, \forall a,b \in A.$

   Ex:- Let IN = Set of natural numbers. Again, let R is a relation defined on IN set R = {(a,b): a is a divisor of b for all a, b, c IN}

   Here R is a anti symmetric relation since if a|b and b|a $\Rightarrow a = b, \forall a,b \in IN.$

4. **Transitive relation** :- A relation R on a set A is called transitive relation if aRb, bRc $\Rightarrow aRc \forall a,b,c \in A.$

   Ex:- A= {1,2,3}

   R = {(1,1), (2,2), (3,3), (1,2), (2,1)}

5. **Identity relation or diagonal relation**:- Let A be a set. Now a relation R on A is called identity if $(a,a) \in R \ \forall a \in A$ and $(a,b) \notin R$ if $a \neq b$.

   Ex: A = {1, 2, 3}

   R = {(1,1), (2,2), (3,3)}.

Note:- An identity relation is also a reflexive relation but reflexive relation may or may not be an identity.

Ex:- A={1, 2, 3}

R_1 = {(1,1), (2,2), (3,3)} reflexive

I = { (1,1), (2,2), (3,3)} Identity as well as reflexive.

(Equivalence Relation):- Let A be a set. Now a relation R on A is called equivalence relation if the following conditions are satisfied:-

(i) R is reflexive i.e. $aRa \forall a \in A$.

(ii) R is symmetric i.e. $aRb \Rightarrow bRa \forall a, b \in A$

(iii) R is transitive i.e. $aRb, bRc \Rightarrow aRc \Rightarrow \forall a, b \in A$

Ex:- Let S=set of all triangles in a plane. Now the 'relation of congruency' i.e. '$\simeq$'on S is an equivalence relation. Since $\Delta_1 \simeq \Delta_1$; $\Delta_1 \simeq \Delta_2 = \Delta_2 \simeq \Delta_1$; $\Delta_1 \simeq \Delta_2$ and $\Delta_2 \simeq \Delta_3$

$= \Delta_1 \simeq \Delta_3, \forall \Delta_1, \Delta_2, \Delta_3 \in S$.

(Partial order relation):- Let A be a set. Now a relation A is called partial relation if R is (i) Reflexive :- (a, a) $\in R \; \forall a \in A$.

(ii) Anti Symmetric :- If aRb and bRa = a=b

(iii) Transitive :- If aRb, bRc = aRc.

The set A together partial order relative $' \leq '$ is called partially ordered set and is denoted by $(A, \leq)$.

Example of partial order relation: - Let A = $\mathbb{Z}$ (Set of integers). R = a relation of "greater than or equal to" on set of integers is a partial order relation. Again $(\mathbb{Z}, \geq)$is a partially ordered set.

(Total order relation):- Let A be a set. Now a relation R on A is called total order relation or linear order relation or a chain if : -

(i) R is a partial order relation on A.

(ii) Either $a \leq b$ or $a \geq$b $\forall\ a, b \in A$.

If there exists a total order relation on A, then the set A with total order relation R, is called totally ordered set.

Ex:- The set of real numbers IR with the relation '$\leq$'(less than or equal to), is a totally ordered set and '$\leq$' on IR is a total order relation since the following conditions are satisfied :-

(i) $x \leq x \forall x \in IR$ reflexive

(ii) $x \leq y$ and y $\Rightarrow x = y$ Anti Symmetric

(iii) $x \leq y, y \leq \mathbb{Z} \Rightarrow x \leq \mathbb{Z}$ Transitive

(iv) $for\ every\ real\ numbers\ x, y \in IR\ either\ x \leq y\ or\ y \leq x. It\ is\ the\ comparability.$

Note :- The above example is also partial order relation.

(Distinguish between partial order relation or total order relation by construction an example of a partially ordered set which is not totally ordered):-

By the definition of total order relation, it is clear that ever total order relation is a partial order relation. To prove that the converse is not true, we construct the following example;

Let IN = set of natural numbers.

= {1, 2, 3, ………………}.

Let $a, b \in IN$ and $a \leq$ b means a divides b. Then (IN, $\leq$) is a partially ordered set which is not totally ordered. Because the followings are hold true:-

(i) $a \leq a \; \forall \; a \in IR$ since a divides a

(ii) a $\leq b \;\&\; b \leq a$ y $\Longrightarrow a = b \; (a, b \in IN)$ Since if a divides b & b divides a then a=b.

(iii) $a \leq b \; and \; b \leq c \Longrightarrow a \leq c \; (a, b, c \; \in IN)$

Since if a divides b & b divides C then a divides C. therefore the relation ≤ is (i) reflexive, (ii) anti symmetric & (iii) transitive on IN, so ≤ is a partial ordered relation on IN, thus (IN, ≤) is a partially ordered set.

Now, let 2, 5 ∈ IN. 2 does not devide 5 and also 5 does not divide 2.

Therefore, ≤ is not a total order relation on IN thus (IN, ≤) is a partially ordered set which is not totally ordered. However, there are many totally ordered subsets of (N, ≤), for ex: the subsets {2, 4, 12}, {5, 10, 20}, (2, 2^2, 2^3…………..) of IN with induced partial order relation are totally ordered set.