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Langevin's theory of Paramagnetism -

Consider a molecular gas containing N molecules or atoms per unit volume at a temperature T. Each atom or molecule is assumed to possess a permanent magnetic moment **m**. If all the dipoles are along the applied magnetic field, the magnetization **M** will be $N\mathbf{m}$. Actually the dipoles are pointing in all directions. The magnetic potential energy of the dipole, which is inclined at an angle θ with the applied field **B**, is given by

$$U = -\mathbf{m}\cdot\mathbf{B} = -mB\cos\theta. \qquad ...(1)$$

According to the Boltzmann distribution law, the relative probability of finding a dipole making an angle θ with the **B** – direction is $e^{-U/kT}$, where k is the Boltzmann constant. Thus the number of dipoles dN, out of total number N, inclined between angles θ and θ + $d\theta$ with the magnetic field **B** will be given by

$$dN \propto e^{-U/kT}\,d\Omega$$
$$= C\,e^{-U/kT}\,d\Omega, \qquad ...(2)$$

where C is a constant of proportionality and $d\Omega$ is the solid angle subtended by the cones of semi-vertical angles θ and θ + $d\theta$ and given by

$$d\Omega = 2\pi\sin\theta\,d\theta$$

$$\therefore \quad dN = C e^{-U/kT}\cdot 2\pi\sin\theta\,d\theta = C\cdot 2\pi e^{mB\cos\theta/kT}\sin\theta\,d\theta.$$
$$= 2\pi C\,e^{\alpha\cos\theta}\sin\theta\,d\theta, \qquad ...(3)$$

where $\alpha = mB/kT$.

Integration of Eq. (103) will give the total number of dipoles per unit volume N. Thus

$$N = \int_0^\pi 2\pi C e^{\alpha\cos\theta}\sin\theta\,d\theta = \frac{4\pi C\sinh\alpha}{\alpha}$$

or

$$C = N\alpha/4\pi\sinh\alpha. \qquad ...(4)$$

$$\therefore \quad dN = \frac{N\alpha}{2\sinh\alpha}e^{\alpha\cos\theta}\sin\theta\,d\theta. \qquad ...(5)$$

The component of each dipole moment parallel to **B** is $m\cos\theta$, hence the total magnetic moment per unit volume

$$M = \int_0^{\pi} m\cos\theta\, dN = \frac{mN\alpha}{2\sinh\alpha}\int_0^{\pi} e^{\alpha\cos\theta}\sin\theta\cos\theta\, d\theta$$

$$= \frac{mN\alpha}{2\sinh\alpha}\left[\frac{e^{\alpha\cos\theta}}{\alpha^2} - \frac{\cos\theta\, e^{\alpha\cos\theta}}{\alpha}\right]_0^{\pi}$$

$$= Nm\,(\coth\alpha - 1/\alpha). \qquad ...(6)$$

Since Nm is the saturation value of the magnetic moment, say M_0, hence

$$M = M_0\,(\coth\alpha - 1/\alpha) = M_0 L(\alpha). \qquad ...(7)$$

This bracket term ($\coth\alpha - 1/\alpha$) is known is *Langevin function, $L(\alpha)$*. It is plotted as a function of α, in Fig. 13.25. This curve indicates that for large values of α the function tends to unity, *i.e.,* $M \rightarrow M_0$. It is the case when all the atomic dipoles are parallel to **B**. For small values of α, the curve is linear, as for smaller values of α the Langevin function

$$L(\alpha) = \coth\alpha - 1/\alpha = \alpha/3 = mB/3kT.$$

For most cases of paramagnetics, $\alpha << 1$ at ordinary temperature even for **B** as large as 1 tesla, therefore **M** can be approximated by the lower values of α, as

$$\mathbf{M} = \frac{M_0\, m\mathbf{B}}{3kT} = \frac{Nm^2\mathbf{B}}{3kT}. \qquad ..(8)$$

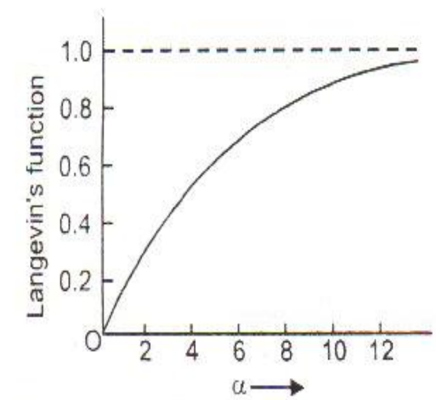
For a paramagnetic substances, **M** is very small positive value, hence $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \approx \mu_0 \mathbf{H}$. Therefore, we have

$$\mathbf{M} = Nm^2\mu_0\,\mathbf{H}/3kT. \qquad ...(9)$$

For small values of α and thus for higher values of T

$$\text{Susceptibility} \quad \chi = \frac{M}{H} = \frac{M_0(\coth\alpha - 1/a)}{H} = \frac{m^2N\mu_0 H}{3kTH}$$

$$= \frac{\mu_0 M_0^{\,2}}{3NkT} = \frac{\text{const}}{T}. \qquad ...(10)$$



Variation of Langevin function.

Here the constant is called the *Curie constant* and this relation is known as *Curie law*. Curie law is well verified experimentally and is very reasonable as at higher temperatures the thermal agitation opposes the dipole alignments along the **B**-direction. It breaks down at low temperatures. The temperature below which this law ceases can reach as low as 1.3^0K, for some paramagnetic substances. At a very low temperature $\alpha >> 1$ and $M = Nm$, which is independent of magnetic field **B**. Thus M takes its saturation value M_0.