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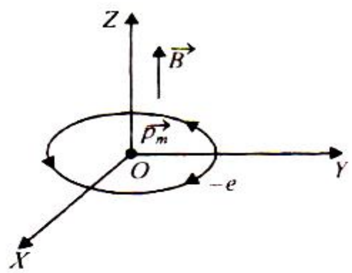
Langevin's theory of diamagnetism - An electron revolving in a circular orbit around the nucleus in an atom is equivalent to a current loop. If $-e$ is the charge on the electron and ω its angular velocity in the orbit, then the equivalent current

$$I = \frac{\text{Charge}}{\text{Time taken to complete the orbit}} = \frac{-e}{2\pi/\omega} = \frac{-e\omega}{2\pi}$$

Electron revolving in counter clockwise direction. Suppose the electron revolves in a circular orbit of radius r in a counter clockwise direction in the $X-Y$ plane, with the nucleus at the origin, then the magnetic moment of the electron

$$\vec{p}_m = I\vec{a} = \frac{-e\omega}{2\pi}\pi r^2\hat{k} \qquad [\because \text{ Area vector } \vec{a} \text{ is along } +Z \text{ axis}]$$

$$= \frac{-e\omega r^2\hat{k}}{2}$$



The direction of dipole magnetic moment $\vec{p}_m$ is along the $-Z$ axis.

As the electron is moving in a circular orbit, the centripetal force acting on it

$$F = m\omega^2 r \qquad ...(i)$$

where m is the mass of the electron. The direction of this force is *radially* inward and it is provided by the *electrostatic force of attraction* between the nucleus and the electron.

If an external magnetic field $\vec{B}$ is now appied perpendicular to the plane of the orbit of this single electron *i.e.*, in the $+Z$ direction, then an additional magnetic force ΔF will act on it, given by

$$\Delta\vec{F} = e\,\vec{v}\times\vec{B}$$

Since $\vec{B}$ is perpendicular to $\vec{v}$, the magnitude of the force

$$\Delta F = evB = er\omega B = Ber\omega \qquad ...(ii)$$

This force will also act *radially inward* on the electron at right angles to both the orbit and the magnetic field. Thus the force on the electron due to the external magnetic field and the electrostatic force acting on it due to the nucleus both act in the same direction.

$\therefore$ Net force acting on the electron in the magentic field

$$= -(m\omega^2 r + Ber\omega)$$

Under the effect of this increased force the electron can either move in an orbit of smaller radius or have a greater velocity in the same orbit or both. But according to the laws of quantum mechanics which are applicable to the electronic orbits, the electron can move only in specific orbits of fixed radii governed by quantum laws. Hence the only alternative for the electron is to increase its angular velocity in the original orbit. If $\omega + \Delta\omega$ is the angular velocity of the electron after the application of the magnetic field, the new magnetic moment of the electron

$$\vec{p}_m + \Delta\vec{p}_m = -\frac{er^2}{2}(\omega + \Delta\omega)\hat{k}$$

$\therefore$ Change in the magnetic moment

$$\Delta\vec{p}_m = -\left[\frac{er^2}{2}(\omega + \Delta\omega) - er^2\omega\right]\hat{k}$$

$$= -\frac{er^2}{2} \Delta\omega \hat{k} \quad \text{...(iii)}$$

According to relation (*i*) $F = m\omega^2 r$

$$\therefore \quad \Delta F = 2mr\omega \, \Delta \omega \quad \text{...(iv)}$$

From relation (*ii*) $\Delta F = Ber\omega$

$$\therefore \quad 2mr\omega\Delta\omega = Ber\omega$$

or

$$\Delta\omega = \frac{e}{2m} B$$

Substituting the value of $\Delta\omega$ in (*iii*), we have

$$\Delta \vec{p}_m = \frac{-e^2r^2}{4m} B \hat{k}$$

$$= \frac{-e^2r^2}{4m} \vec{B} \quad \text{...(v)}$$

The negative sign shows that the change in magnetic moment takes place in a direction opposite to $\vec{B}$. Thus the electron develops an induced magnetic moment $\Delta \vec{p}_m$ in a direction opposite to the external magnetic field.

The change in magnetic moment of the orbiting electron is $\frac{-e^2r^2}{4m} \vec{B}$.

Electron revolving in clockwise direction. If we consider the electron as moving in the *clock-wise* direction, the area vector of the current loop

$$\vec{a} = -\pi r^2 \hat{k}$$

and the magnetic moment of the electron before the application of the magnetic field is

$$\vec{p}_m = \left(-\frac{e\omega}{2\pi}\right)\left(-\pi r^2 \hat{k}\right) = \frac{e\omega r^2}{2} \hat{k}$$

The directon of $\vec{p}_m$ is, therefore, along + Z axis.

As in this case the direction of the velocity vector has also changed, the force due to the external magnetic field will now act on the electron in the *radially outward* direction *i.e.*, opposite to that of the electrostatic force. The net force on the electron will decrease and the electron will move in the same orbit with a smaller angular velocity $(\omega - \Delta\omega)$. The change in the magnetic moment will, therefore be given by

$$\Delta \vec{p}_m = \left[\frac{er^2}{2}(\omega - \Delta\omega) - \frac{er^2}{2}\omega\right]\hat{k}$$

$$= -\frac{er^2}{2} \Delta \omega \hat{k} \text{ ...(v) (a)}$$

which on substituting the value of $\Delta\omega = \frac{e}{2m} B$ gives

$$\Delta \vec{p}_m = \frac{-e^2 r^2}{4m} \vec{B} \quad \text{...(v) (a)}$$

Thus the value of $\Delta \vec{p}_m$ is the same in direction as well as in magnitude whether the electron is moving in the clockwise or anticlockwise direction.

If N is the number of electrons per unit volume and Z the atomic number, then

$$\vec{P}_m = \frac{-N Z e^2 r^2}{4m} \vec{B} = \frac{-N Z e^2 r^2 \mu_0}{4m} \vec{H}$$

and susceptibility $$\chi_m = \chi_m = \frac{\vec{P}_m}{\vec{H}} = \frac{-N Z e^2 r^2 \mu_0}{4m}$$

In the expression, (v) (a) if the electronic orbit is not perpendicular to the applied field, then r is replaced by r_1, the projection of the radius r of the orbit on the plane perpendicular to the magnetic field *i.e.*, in the $X - Y$ plane.

If x, y, z are the co-ordinates of the radius r of the orbit, then

$$r^2 = x^2 + y^2 + z^2$$

and $$r_1^2 = x^2 + y^2$$

If the radii are arranged in all possible directions, then the average value of

$$r^2 = x^2 + y^2 + z^2 = 3x^2$$

and $$r_1^2 = x^2 + y^2 = 2x^2 = \frac{2}{3} r^2$$

If we denote the average value of r^2 as $< r^2 >$, we have relation (v) as

$$\Delta \vec{p}_m = \frac{-e^2}{4m} \cdot \frac{2}{3} < r^2 > \vec{B}$$

$$= \frac{-e^2}{6m} < r^2 > \vec{B}$$

If N is the number of electrons per unit volume and Z the atomic number, then the diamagnetic dipole moment per unit volume

$$\vec{P}_m = \frac{-NZe^2}{6m} < r^2 > \vec{B} = \frac{-NZe^2}{6m} < r^2 > \mu_0 \vec{H}$$

$$= \chi_d \vec{H} \qquad ...(vi)$$

where χ_d is the diamagnetic susceptibility of the material.

$$\therefore \qquad \chi_d = -\frac{NZe^2\mu_0}{6m} < r^2 > \qquad ...(vii)$$

According to the laws of atomic structure, the electrons in an atom have a tendency to exist in pairs with their angular momentum vectors pointing in opposite directions. Thus for most of the atoms which have an even number of electrons, the total angular momentum is zero. Such atoms have zero magnetic dipole moment and are *diamagnetic*.