Dr. Manoj Kumar, M.SC, M.Phil, Ph.D
Deptt. of Mathematics, MMC, Patna,
Email Id: kumarmanojyadav9@gmail.com
Contract No: 9572487276

Programme -01 for B. Sc (Hons) Part-1

Fundamental of Set

Set: - A Well-defined class is called a set. By the word Well-Defined we mean two things as (i) distinct objects
(ii) distinguishable objects.

The objects which constitute the Set is called elements or members of the set. Generally we denote the set by the capital letters of English alphabet and the elements of the set are denoted by small letters of English language or numbers.

Ex: - 1. A $=\{a, e, i, o, u\}$

Ex: - 2. IN $=\{x: x \text{ is a natural number}\}$

Ex: - 3. B $=\{x: x \text{ is a student of } B.Sc \text{ Matshs } (H) \text{of MMC College, Patna}\}$

$(Various\ types\ of\ set):-$

1. Null Set or empty set or avoid set :-
   A set having no element is called a hull set. It is generally denoted by φ.
   Thus $\varphi = \{\dots \dots .\}$.
2. Singleton set :- A set having only-one element is called singleton set.
   Ex :- X $=\{O\}$
3. Pair Set :- A set having only two elements is called pair set.
   Ex :- Y $= \{x: x^2 - 5 = 0\}$
4. Finite Set :- A set having finite no of elements is called finite set. That is if the counting of elements of a set comes to an end, the set is called finite set.
   Ex :- Z $= \{1,2,3,4,5, \dots \dots \dots \dots \dots \dots \dots ..100\}$
   P $= \{x: x \text{ is a student of } B.Sc \text{ maths } (H) \text{of Patna universty.}\}$

5. Infinite set: - A set having infinite no of elements is called infinite set.

Ex :- IN = $\{1,2,3,4,5, \dots \dots \dots \dots \dots \dots \dots ..\}$

Ex :- $\mathbb{Z}$ = $\{ \dots \dots, -3, -2, -1, 0, 1, 2, 3 \dots \dots\}$

(Subset) :- Let A and B be the two sets such that each elements of A is also an elements of B, then A is called subset of B and is denoted by $A \subseteq B$.

Ex :-1. A = $\{1,2,3,4,5\}$

IN = $\{1,2,3,4, \dots \dots \dots \dots \dots \dots \dots ..\}$

$\Rightarrow A \subseteq IN$

Ex :-2. $X = \{x : x \text{ is a student of MMC Patna}\}$

Y = $\{x : x \text{ is a student of Patna university}\}$

$\Rightarrow X \subseteq Y.$

(Proper Subject) :- Let A and B be the two sets such that each elements of A is also an element of B but B has at least one element which is not in A then A is called proper subset of B and is written as $A \subset B$. Again B is called super set of A.

Ex :- 1. A = $\{1,2,3\}$

B = $\{1,2,3,4,5\}$

$A \subset B$.

Ex :- 2. X = $\{a, e, i, o, u\}$

Y = $\{a, b, c, \dots \dots \dots \dots \dots x, y, z\}$

$X \subset Y$.

(Equality of sets) :- If $A \subseteq B$ and $B \subseteq A$ then A and B are called equal sets and the relation between A and B is called equality of sets and is written as A= B.

Ex :- A =$\{a, e, i, o, u\}$

B = $\{i, e, q, o, u\}$

A=B.

(Power Set) :- Let A be a set. Now the collection of all the possible subsets of A denoted by P (A), is called power set of A. Ex :- A =$\{1,2,3\}$

P (A) = power set of A $\{\varphi, \{1,2,3\}, \{1\}, \{2\}, \{3\}, \ \{1,2\}\{1,3\}, \{2,3\}\}$

(Universal Set):- Whenever we make a discussion over the sets then the set among then, which is the super set of all the remaining sets. Is called universal set, it is denoted by X,E, U or Ω.

Ex: - $\quad$ $A = \{1,2\},\ B = \{2,3,4,5\}, C = \{1,3,5,7,9\}$

X = $\{1,2,3, \dots \dots \dots \dots \dots 10\}. X\ is\ universal\ set.$

(Comparable sets) :-

If A and B are two sets such that either A ⊆ B or B ⊆ A then A and B are called comparable sets. Ex: - 1. A = $\{1,2,3\}$, B = A=$\{1,2,3,4,5\}$ A ⊆ B . A and B are comparable sets.

Ex : 2 $\quad$ A = $\{a, b, c, d, \dots \dots \dots \dots \dots \dots x, y, z\}$

B = A $\{a, e, i, o, u\}$

B ⊆ A ⇒ A and B are comparable sets.

(Disjoint Sets) :-

If A and B are two sets such that A and B have no element in common than A and B are called disjoint sets.

Ex : -1. $\quad$ A = $\{1,3,5\}, B = \{2,4,6\}$.

There A and B have no elements in common and therefore A and B disjoint sets. If A and B are not disjoint sets then they are called joint sets. Then they are called joint sets.

(Difference of sets) :- If A and B be the two sets then (A-B) is called difference of A B and is denoted by A-B and is defined as

(A-B) = $\{x: x \in A\ but\ x \notin B\}$.

Slly, B-A = difference of B and A = $\{y: y \in B \text{ but } y \notin A\}$

Ex :- A = $\{1,2,3,4,5\},\ B = \{2,4,5,6,7,8,9\}$

A-B = $\{1,3\},\quad B - A = \{6,7,8,9\}$

Moreover A-B = A - $(A \cap B)$

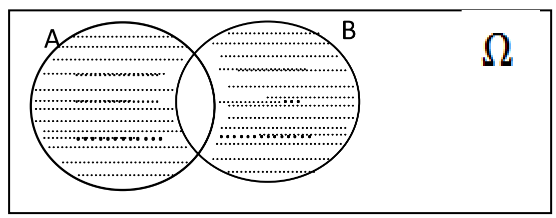
B-A = B - $(A \cap B)$

(Symmetric Difference of two sets):-

Let A and B are two sets. Now the symmetric difference of A and B denoted by $A \triangle B$ is defined as $A \triangle B = \{x : x \in (A - B) or\ x \in (B - A)\}$

That is $A \triangle B = (A - B) \cup (B - A)$

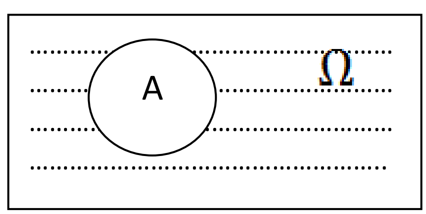
Pictorially, $A \triangle B$



(Complementary set or complement of a set) :-

Let A be a set and Ω be the universal set now the complement of A denoted by A^c OR A' is defined as $A^c = \Omega$ - A = $\{x: x\epsilon\Omega \text{ but } x \notin A\}$

Pictorially, A^c



(Union of sets) :- Let A and B be the two sets then the union of A and B denoted by A ∪ B is defined as A ∪ B = $\{x : x\epsilon A \text{ or } x\epsilon B\}$.

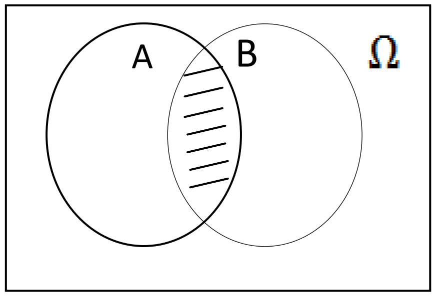
Slly $A \cup B \cup C \ldots\ldots\ldots \cup \mathbb{Z} = \{x: x\epsilon A \text{ or } x\epsilon B \text{ or } \ldots\ldots \text{ or } x\epsilon z\}$.

Ex:-1. A= $\{a, e\},\quad B = \{b, x, d, e\}$

A ∪ B = $\{a, b, c, d, e\}$

(Intersection of sets) :- Let A and B be the sets. Now the intersection of A and B denoted by A⋂B⋂C⋂. ….⋂$\mathbb{Z}$ = $\{x: x\epsilon A \ and \ x\epsilon B \ and \ldots \ldots .. x\epsilon z\}$

Pictorially . A⋂B



(Indexed Family of sets) :- Let I be a fixed but arbitrary non-empty set with member $i \in I$ and let there be defined a set $A_i, \forall \ i \in I$, then the family of sets A_i is called and indexed family of sets and the set I is called an indexed set. Symbolically indexed family of sets can be written as $\{A_i : i\epsilon I\} \ or \ \{A_i\}_{i\epsilon I}$ where I ⊆ IN $(i.e. I \ is \ the \ subset \ of \ set \ of \ natural \ number \ IN)$

(Union and intersection of indexed family of sets):- Let $\{A_i : i\epsilon I \ (\subseteq IN)\}$ be an indexed family of subsets of a universal set Ω. Then the union of family of sets is denoted by $\underset{i\epsilon I}{\cup A_i} = \{x: x\epsilon A_i \ for \ at \ least \ on \ i\epsilon I\}$

The intersection of the family of the sets is denoted by $\underset{i\epsilon I}{\cap A_i}$ and is defined as

$\underset{i\epsilon I}{\cap A_i} = \{x: x\epsilon A_i for \ all \ i\epsilon I\}$.

Dr. Manoj Kumar, M.SC, M.Phil, Ph.D
Deptt. of Mathematics, MMC, Patna,
Email Id: kumarmanojyadav9@gmail.com
Contract No: 9572487276

Programme -02 for B.Sc (Hons) Part-1

Generalised form of De Margan's Laws:-

The general form of De Margan's Laws are given as the followings:-

1. $\left(\underset{i \in I}{\cup} A_i\right)^c = \underset{i \in I}{\cap} A^c{}_i$

That is the employment of the union of sets is the union of their complements.

Prof :- Let $x \in \left(\underset{i \in I}{\cup} A_i\right)^c$. That is

$$x \in (A_1 \cup A_2 \cup A_3)^c$$

$$\Rightarrow x \notin (A_1 \cup A_2 \cup A_3 \ldots \ldots \ldots \ldots)$$

$$\Rightarrow x \notin A_1 \, and \, x \notin A_2 \; and \; x \notin A_3$$

$$\Rightarrow x \in A^c{}_1 \; and \, x \in A^c{}_2 \; and \, x \in A^c{}_3 \ldots \ldots \ldots \ldots$$

$$\Rightarrow x \in (A^c{}_1 \cap A^c{}_2 \cap A^c{}_3 \ldots \ldots \ldots \ldots)$$

$$\Rightarrow x \in \left(\underset{i \in I}{\cup} A_i\right)^c$$

So by the definition of subset, we can write

$$\left(\underset{i \in I}{\cup} A_i\right)^c \subseteq \left(\underset{i \in I}{\cup} A_i\right)^c \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (i)$$

Again, let $y \in \left(\underset{i \in I}{\cap} A^c{}_i\right)$.

$$\Rightarrow y \in (A^c{}_1 \cap A^c{}_2 \cap A^c{}_3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots)$$

$$\Rightarrow y \in A^c{}_1 \text{ and } y \in A^c{}_2 \text{ and } y \in A^c{}_3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$\Rightarrow y \notin A_1 \text{ and } y \notin A_2 \text{ and } y \notin A_3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$\Rightarrow y \notin (A_1 \cup A_2 \cup A_3 \ldots \ldots \ldots \ldots)$$

$$\Rightarrow y \notin \left(\underset{i \in I}{\cup} A_i\right) \Rightarrow y \in \left(\underset{i \in I}{\cup} A_i\right)^c$$

SO, $\left(\underset{i \in I}{\cap} A^c{}_i\right) \subseteq \left(\underset{i \in I}{\cup} A_i\right)^c \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (ii)$

From (i) and (ii), $\left(\bigcup_{i \in I} A_i\right)^c = \bigcap_{i \in I} A^c{}_i$. Hence the proof.

2. To $\left(\bigcap_{i \in I} A_i\right)^c = \bigcup_{i \in I} A^c{}_i$; $I \subseteq IN$.

Let $x \in \left(\bigcap_{i \in i} A_i\right)^c$.*That is*

$x \in (A_1 \cap A_2 \cap A_3 \ldots\ldots\ldots)^c$.

$\Rightarrow x \notin (A_1 \cup A_2 \cup A_3 \ldots\ldots\ldots)$

$\Rightarrow x \notin (A_1 \; or \; x \notin A_2 \; or \; x \notin A_3 \ldots\ldots\ldots)$

$\Rightarrow x \in A^c{}_1 \; or \; x \in A^c{}_2 \; or \; x \in A^c{}_3 \ldots\ldots\ldots$

$\Rightarrow x \in (A^c{}_1 \cup A^c{}_2 \cup A^c{}_3 \ldots\ldots\ldots)$

$\Rightarrow x \in \left(\bigcap_{i \in I} A_i\right)^c \Rightarrow \left(\bigcap_{i \in I} A_i\right)^c \subseteq \left(\bigcup_{i \in I} A^c{}_i\right) \ldots\ldots\ldots (i)$

Again , Let $y \in \bigcap_{i \in I} A^c{}_i$

$\Rightarrow y \in (A^c{}_1 \cup A^c{}_2 \cup A^c{}_3 \ldots\ldots\ldots\ldots\ldots)$

$\Rightarrow y \in A^c{}_1 \text{ or } y \in A^c{}_2 \text{ or } A^c{}_3 \text{ or}$

$\Rightarrow y \notin A_1 \text{ or } y \notin A_2 \text{ or } y \notin A_3 \text{ or}$

$\Rightarrow y \notin (A_1 \cap A_2 \cap A_3 \cap \ldots\ldots\ldots)$

$\Rightarrow y \in (A_1 \cap A_2 \cap A_3 \cap \ldots\ldots\ldots)^c$

$\Rightarrow y \in \left(\bigcap_{i \in I} A_i\right)^c$

So, $\left(\bigcup_{i \in i} A^c{}_1\right) \subseteq \left(\bigcap_{i \in i} A_i\right)^c \ldots\ldots\ldots (ii)$

Therefore, by the definition of equality of sets, from (i) and (ii), we get

$\left(\bigcap_{i \in I} A_i\right)^c = \bigcup_{i \in I} A^c{}_i$. Hence the proof.

(Second form of De Morgan's Law) :-

Let A ,B, X be the three sets. Then

1. $X - (A \cup B) = (X - A) \cap (X - B)$

2. $X - (A \cap B) = (X - A) \cup (X - B)$

Proof - Let $x \in X - (A \cup B)$

$\Rightarrow x \in X \, and \, x \notin (A \cup B).$

$\Rightarrow x \in X \, and \, (x \notin A \, and \, x \notin B).$

$\Rightarrow (x \in X \, and \, x \notin A) \, and \, (x \in X \, and \, x \notin B)$

$\Rightarrow x \in (X - A) \cap (X - B)$

So, by the definition of subset, we can write

$$X - (A \cup B) \subseteq (X - A) \cap (X - B) \ldots \ldots \ldots \ldots \ldots \ldots (i)$$

Again, let $y \in (x - a) \cap (x - B)$

$\Rightarrow y \in (X - A \, and \, Y \in (X - B)$

$\Rightarrow (y \in X \, and \, y \notin A) \, and \, (y \in X \, and \, y \notin B)$

$\Rightarrow y \in X \, and \, (y \notin A) \, and \, (y \notin B)$

$\Rightarrow y \in X \, and \, y \notin (A \cup B)$

$\Rightarrow y \in X - (A \cup B)$

$\Rightarrow (X - A) \cap (X - B) \subseteq (X - A \cup B) \ldots \ldots \ldots \ldots \ldots (ii)$

From (i) and (ii), and by the definition of equality of sets, we have

$X - (A \cup B) = (X - A) \cap (X - B).$ *Hence the proof.*

Proof of (2) Let $x \in (X - A \cap B).$

$\Rightarrow x \in X \, and \, x \notin (A \cap B)$

$\Rightarrow x \in X \, and \, (x \notin A \, or \, x \notin B)$

$\Rightarrow (x \in X \, and \, x \notin A) \, or \, (x \in X \, and \, x \notin B)$

$\Rightarrow x \in (X - A) \, or \, x \in (X - B)$

$\Rightarrow x \in (X - A) \cup (X - B)$

Therefore, by the definition of subset, we can write,

$$X - (A \cap B) \subseteq (X - A) \cup (X - B) \ldots\ldots\ldots\ldots\ldots \quad (i)$$

Again, let $y \in (X - A) \cup (X - B)$

$\Rightarrow y \in (X - A) or \; y \in (X - B)$

$\Rightarrow (y \in X \, and \, (y \notin A) \, or \, (y \in X \, and \, y \notin B)$

$\Rightarrow y \in X \, and \, (y \notin (A \cap B)$

$\Rightarrow y \in X - (A \cap B)$ ………………………………………. (ii)

From (i) and (ii) , and by the definition of equality of sets, we have,

$$X - (A \cap B) = (X - A) \cup (X - B) \quad Hence\ the\ proof.$$

Theorem 1: If $\{A_i : i \in I\}$ *be an indexed faimly of subsets of the universal set* Ω*, then the general distributive lawas are given as* :-

(i) $B \cup \left(\bigcap_{i \in I} A_i \right) = \bigcap_{i \in I} (B \cup A_i)$

(ii) $B \cap \left(\bigcup_{i \in I} A_i \right) = \bigcup_{i \in I} (B \cap A_i)$

Proof of (i) :- Let $x \in B \cup \left(\bigcap_{i \in I} A_i \right)$

$\Rightarrow x \in B \text{ or } x \in \left(\bigcap_{i \in I} A_i \right) for\ \ i \in I$

$\Rightarrow x \in B \cup A_i \ \ \forall\ i \in I$

$\Rightarrow x \in \bigcap_{i \in I} (B \cup A_i) \Rightarrow$

So, by the definition of subset, we can write

$$B \cup \left(\bigcap_{i \in I} \right) \subseteq \bigcap_{i \in I} (B \cup A_i) \ldots\ldots\ldots.(i)$$

Slly we can prove that

$$\bigcap_{i \in I} (B \cup A_i) \subseteq B \cup \left(\bigcap_{i \in I} A_i \right) \ldots\ldots\ldots(ii)$$

From (i) & (ii), and by the define of equality of sets, we have

$$B \cup \left(\bigcap_{i \in I} A_i \right) \subseteq \bigcap_{i \in I} (B \cup A_i) \quad \text{Proved.}$$

(ii) Let $x \in B \cap \left(\bigcap_{i \in I} A_i \right)$.

$\Rightarrow x \in B \text{ and } (x \in A_i, \text{for at least one } i \in I)$

$\Rightarrow (x \in B \text{ and } x \in A_i), \text{for at least one } i \in I)$

$\Rightarrow x \in (B \cap A_i), \text{for at least one } i \in I)$

$\Rightarrow x \in \bigcup_{i} (B \cap A_i),$

So, $B \cap \left(\underset{i \in I}{\cup} A_i \right) \subseteq \underset{i \in I}{\cup} (B \cap A_i)$ .... (1)

Slly $\underset{i \in I}{\cup} (B \cap A_i) \subseteq B \cap \left(\underset{i \in I}{\cup} A_i \right)$ ....(2)

From (1) & (2), $B \cap \left(\underset{i}{\cup} A_i \right) = \underset{i \in I}{\cup} (B \cap A_i)$. Proved .

Dr. Manoj Kumar, M.SC, M.Phil, Ph.D

Deptt. of Mathematics, MMC, Patna,

Email Id: kumarmanojyadav9@gmail.com

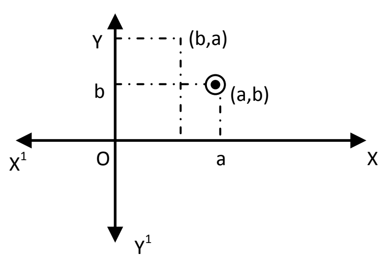
Contract No: 9572487276

Programme -03 for B.Sc (Hons) Part-1

Cartesian Product of SETS

(Ordered Pair) :- Let a and b be the two real numbers. Then the ordered pair of a and b denoted by (a,b) is a composition of x-co-ordinate and Y-co-ordinate.

Generally (a,b) $\neq (b, a)$. Since (a,b) and (b,a) are two different points in the plane.



It is very clear that (a,b)=(c,d) if a=c and b=d. ordered pair is also called 2-tuples.

(Cartesian Product) :- Let A and B are the two sets. Then the Cartesian product of A and B are the two sets. Then the Cartesian product of A and B denoted. By (A × B) is defined as A × B = $\{(a, b) : a \in A, b \in B\}$.

Similarly , B × A = Cartesian product of B and A = $\{(b, a): b \in B \text{ and } a \in A\}$

Clearly (A × B) $\neq (B \times A)$.

Ex.1. Let A= $\{1,2\}, B = \{2,3,4\}, C = \{5,6\}$.

Then (i) A × B = $\{1,2\} \times \{2,3,4\} = \{(1,2), (1,3), (1,4), (2,1)(2,3), (2,4)\}$.

(ii) $A \times C = \{1,2\} \times \{5,6\} = \{(1,5),(1,6),(2,5),(2,6)\}$.

(iii) $(A \cap B) \times C = \{2\} \times \{(5,6) = (2,5),(2,6)\}$

Theorems On Cartesian Product.

Theorem 1 :- If A,B and C be the three sets,

Then (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Proof : (i) Let $(x, y) \in A\, X\, (B \cup C)$ *be any element.*

$\Rightarrow x \in A \text{ and } y \in (B \cup C)$

$\Rightarrow x \in A \text{ and } (y \in B \text{ or } y \in C)$

$\Rightarrow (x \in A \text{ and } y \in B) \text{or} (x \in A \text{ and } y \in C)$

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$.

Therefore by the definition of subset, we have

$$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C) \ldots\ldots\ldots\ldots\ldots\ldots. (i)$$

Similarly we can show that

$$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C) \ldots\ldots\ldots\ldots\ldots \quad (ii)$$

From (i) and (ii), and by the definition of equality of sets, we have

$$A \times (B \cup C) = (A \times B) \cup (A \times C). \quad Proved.$$

(ii) Let $(x, y) \in A \times (B \cap C)$ *by any element.*

$\Rightarrow (x, y) \in \big(A \times (B \cap C)\big)$.

$\Rightarrow x \in A \text{ and } y \in (B \cap C)$

$\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)$

$\Rightarrow (x, y) \in (A \times B) \text{and} (x, y) \in (A \times C)$.

$\Rightarrow (x, y) \in (A \times B) \cap A \times C)$.

Therefore by the definition of subset, we have

$$A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly, we can prove that,

$$(A \times B) \cap (A \times C) \subseteq A \times (B \cap C) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Now from (1) and (2), and by the def in of equality of sets, we have

$A \times (B \cap C) = (\mathrm{A} \times \mathrm{B}) \cap (A \times C)$ Hence the proof.

Theorem 2:- If A,B,C and D be the form sets, then

$$(A \times B) \cap (C \times D) = (A \times C) \times (B \cap D).$$

Proof :- Let $(x, y) \in (A \times B) \cap (C \times D)$ be any element.

$$\Rightarrow (x, y) \in (A \times B) and\ (x, y) \in (C \times D).$$

$$\Rightarrow x \in A\ and\ y \in B\ and\ x \in c\ and\ y \in D)$$

$$\Rightarrow (x \in A\ and\ x \in C\ ) and\ (Y \in B\ and\ Y \in D)$$

$$\Rightarrow x \in (A \cap C) and\ y \in (B \cap D).$$

$$\Rightarrow (x, y) \in (A \cap C) \times (B \cap D).$$

$$\Rightarrow (x, y) \in (A \cap C) \times (B \cap D).$$

Therefore, by the definition of subset, we have

$$(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly, we can also prove, that

$$(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2), and by the equality of sets, we have

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$ Hence the proof.

Theorem -3 :- If A,B,C be the three sets, then $(A-B)\times C =(A\times C)-(B\times C)$.

Proof :- Let $\Rightarrow (x, y) \in (A - B) \times C$ be any element.

$$\Rightarrow x \in (A - B) and\ y \in C$$

$$\Rightarrow (x \in A\ and\ x \notin B) and\ y \in C.$$

$$\Rightarrow (x \in A\ and\ y \in C)\ and\ (x \notin B\ and\ y \in C)$$

$$\Rightarrow (x, y) \in (A \times C)\ and\ (x, y) \notin (B \times C)$$

$$\Rightarrow (x, y) \in (A \times C) - (B \times C)$$

So, lay the definition of subset, we have

$$(A - B) \times C \subseteq (A \times C) - (B \times C) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly, we can prove that

$$(A \times C) - (B \times C) \subseteq (A - B) \times C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Now, from (1) and (2), and by the equality of sets, we have

$$(A - B) \times C = (\cap \times C) - (B \times C).$$ Proved

Theorem 4: Let A,B,C,D be the four sets, then

$$(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D) \cup (A \times D) \cup (B \times C).$$

Proof :- Let $(x, y) \in (A \cup B) \times (C \cup D)$ be any element,

$$\Rightarrow x \in (A \cup B) and\ y \in (C \cup D)$$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (y \in C\ or\ y \in D)$$

$$\Rightarrow (x \in A\ or\ dy \in c)\ or\ (x \in B\ and\ y \in D) or\ (x \in A\ and\ y \in l)$$

$or\ (x \in B\ and\ y \in C)$

$$\Rightarrow (x, y) \in (A \times C) or\ (x, y) \in (B \times D) or\ (x, y) \in (A \times D) or\ (x, y) \in (B \times C)$$

$$\Rightarrow (x, y) \in (A \times C) or\ (x, y) \in (B \times D) or\ (x, y) \in (A \times D) or\ (x, y) \in (B \times C).$$

So, by the definition of subset, we have

$$(A \cup B) \times (C \cup D) \subseteq (A \times C) \cup (B \times D) \cup (\cap \times D) \cup (B \times C) \ldots \ldots \ldots \ldots \ldots (1)$$

Similarly, we can also & low that

$$(A \times C) \cup (B \times D) \cup (A \times D) \cup (B \times C) \subseteq (A \cup B) \times (C \cup D)) \ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2) and the definition of equality of sets, we have

$$(A \cup B) \times\ (C \cup D) = (A \times C) \cup (B \times D) \cup (A \times D) \cup (B \times C)\ \ Hence\ the\ Proof.$$

Theorem 5. For any sets A,B and C.

$$C \times (A - B) = (C \times A) - (C \times B).$$

Proof: - It can be proved in the similar way as the proof the Theorem 3.