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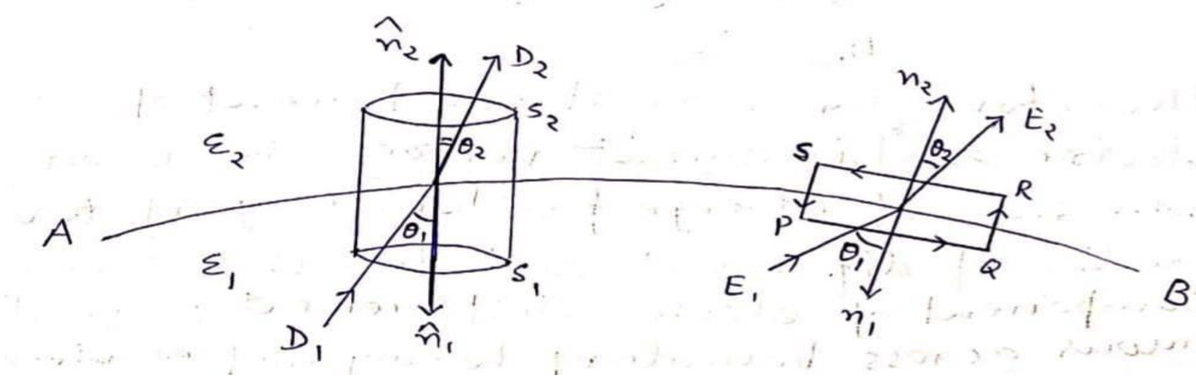
Boundary Conditions at the surface of separation of two dielectric:

Let AB represents a small portion of the boundary of two media of absolute permittivity $\in_1$ & $\in_2$

Consider a small element ds on the boundary which is so small that its curvature may be neglected.

D_1 & D_2 = Electric displacement vectors in the medium on either side of ds

θ_1 & θ_2 = Angle made by D_1 & D_2 with normal



Boundary condition for D –

Consider a small cylinder which intersects the area ds on boundary and its height in very small as compared to diameter of base.

Apply Gauss's law to a small cylinder which intersects the area ds on the boundary

$$\oint \vec{D}.\vec{ds} = \int_c \vec{D}.\vec{ds} + \int_{s_1} \vec{D_1}.\vec{ds} + \int_{s_2} \vec{D_2}.\vec{ds} = q$$

$\because \int_c \vec{D}.\vec{ds} = o$ Because the flux through curved surface of cylinder is very small.

So it can be neglected.

$$O + (\vec{D_1}.\hat{n_1} + \vec{D_2}.\hat{n_2}) = ds = \sigma ds \left[\because \sigma = \frac{q}{ds}\right]$$

$-D_1 \, Cos\theta_1 + D_2 Cos\theta_2 = \sigma$……………………… *(1).*

In majority cases there is no free charge on the boundary . Hence

$-D_1 \, Cos\theta_1 + D_2 Cos\theta_2 = 0$

$D_1 \, Cos\theta_1 = D_2 Cos\theta_2$ ……………………… *(2).*

$D_{1n} = D_{2n}$

Therefore the normal component of electric displacement vector is same on both side of charge free boundary of two media of different dielectric *ie* the normal component of electric displacement is continuous across boundary having no free charges.

Boundary Condition for E-

Consider a small rectangle PQRS of length dl and of negligible height on the boundary with its longest side is parallel to surface of separation in each medium. Work done by unit charge around the rectangle PQRS.

$$\oint \vec{E}.\vec{dl} = \int_{c}^{Q} \vec{E_i}.\vec{dl} + \int_{Q}^{R} \vec{E}.\vec{dl} + \int_{R}^{S} \vec{E_2}.\vec{dl} + \int_{S}^{P} \vec{E}.\vec{dl} = 0$$

$\int_{Q}^{R} \vec{E}.\vec{dl}$. & $\int_{S}^{P} \vec{E}.\vec{dl} = 0$ Because sides QR & SP is very small as compared to PQ & RS

$$\int_{P}^{Q} \vec{E}.\vec{dl} + . \int_{R}^{S} \vec{E_2}.\vec{dl} = 0$$

$$E_1 \sin\theta_1 dl - E_2 \sin\theta_2 dl = 0$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3).$$

$E_{1t}=E_{2t}$

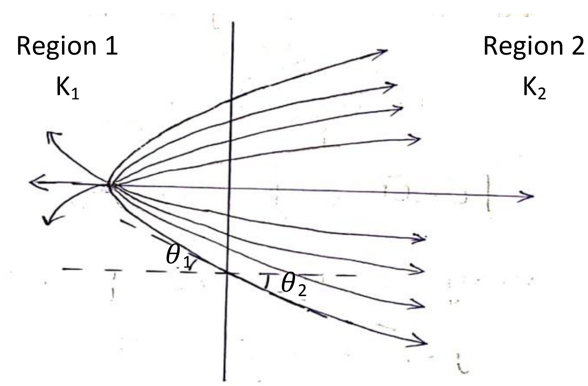
Therefore the tangential component of electric intensities are same on both sides of boundary surface. *ie* the tangential component of electric field is continuous at the boundary between two dielectric

Refraction of electric line of force

Suppose that

$\Theta_1 \& \theta_2 =$ Angle made by electric lines of force with the normal to the surface in region 1 & 2 respectively

Electric displacement vector D = $K \in_0$E will coincide with electric vector.



From eqn (2) and (3)

$$\frac{D_1 \text{Cos } \theta_1}{E_1 \text{Sin } \theta_1} = \frac{D_2 \text{Cos } \theta_2}{E_2 \text{Sin } \theta_2}$$

$$\frac{D_1}{E_1} \text{Cot}\theta_1 = \frac{D_2}{E_2} \text{Cot}\theta_2$$

But $D_1 = K_1 \in_0 E_1$ & $D_2 = K_2 \in_0 E_2$

$$\therefore K_1 \in_0 \text{Cot}\theta_1 = K_2 \in_0 \text{Cot}\theta_2$$

$$K_1 \text{Cot}\theta_1 = K_2 \text{Cot}\theta_2$$

This equation shows that in passing from empty space into a dielectric the electric lines of force or displacement are bent away from normal to the surface.