

M.A. Semester – II
Paper: CC-9: Statistical Methods
Topic: Hypothesis Testing

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Introduction

To test a hypothesis means to tell (on the basis of the data collected) whether or not the hypothesis seems to be valid. In hypothesis testing the main question is: whether to accept the null hypothesis or to reject the null hypothesis? Procedure for hypothesis testing refers to all those steps that are undertaken for making a choice between the two actions i.e., rejection and acceptance of a null hypothesis. The various steps involved in hypothesis testing are stated below:

- i) **Formulation of hypothesis:** The step consists in making a formal statement of the null hypothesis (H_0) and also of the alternative hypothesis (H_a). This means that hypotheses should be clearly stated, considering the nature of the research problem. For instance, Mr. Mohan of the Civil Engineering Department wants to test the load bearing capacity of an old bridge which must be more than 10 tons, in that case he can state his hypotheses as under:

$$\begin{aligned} \text{Null hypothesis } H_0 : \mu &= 10 \text{ tons} \\ \text{Alternative Hypothesis } H_a: \mu &> 10 \text{ tons} \end{aligned}$$

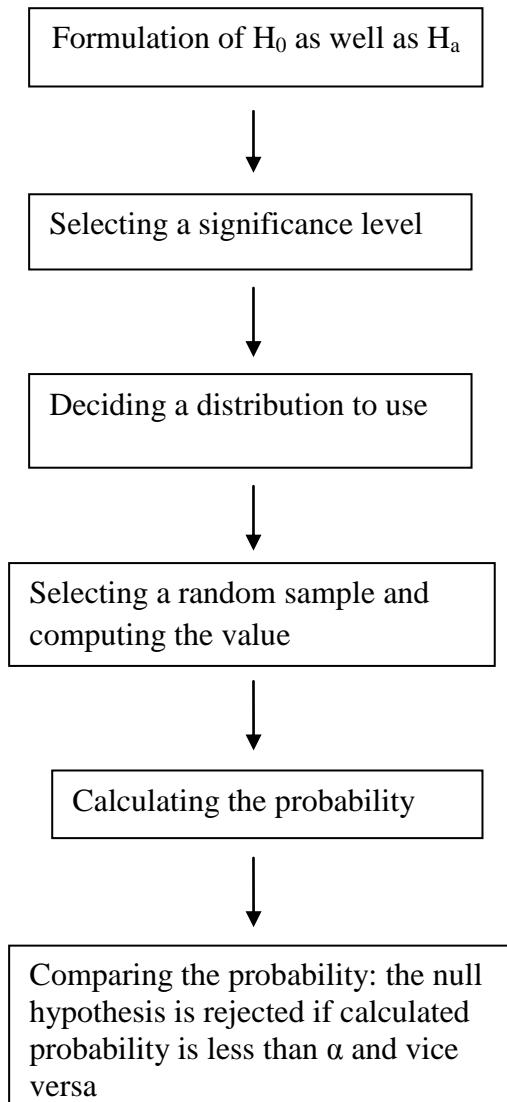
The formulation of hypotheses is an important step which must be accomplished with due care in accordance with the object and nature of the problem under consideration. It also indicates whether one should use a one-tailed test or a two-tailed test. If H_a is of the type greater than ($>$) or of the type lesser than ($<$), we use a one-tailed test, but when H_a is of the type “whether greater or smaller (\neq)” then a two-tailed test is used.

- ii) **Selecting a significance level:** The hypotheses are tested on a pre-determined level of significance and as such the same should be specified. Generally, in practice, either 5% level or 1% level is adopted for the purpose. Significance level normally indicates the chance of committing an error also known as type I error or α . The factors that affect the level of significance are: (a) the magnitude of the difference between sample means; (b) the size of the samples; and (c) the variability of measurements within samples. In brief, the level of significance must be adequate in the context of the purpose and nature of enquiry.
- iii) **Deciding the distribution to use:** After deciding the level of significance, the next step in hypothesis testing is to determine the appropriate sampling

distribution. The choice generally remains between normal distribution and the t-distribution.

- iv) **Selecting a random sample and computing an appropriate value:** Another step is to select a random sample(s) and compute an appropriate value from the sample data concerning the test statistic utilizing the relevant distribution.
- v) **Calculation of the probability:** One has then to calculate the probability that the sample result would diverge as widely as it has from expectations, if the null hypothesis were in fact true.
- vi) **Comparing the probability:** Yet another step consists in comparing the probability thus calculated with the specified value for α , the significance level. If the calculated probability is equal to or smaller than the α value in case of one-tailed test (and $\alpha/2$ in case of two-tailed test), then the null hypothesis is rejected (i.e., accept the alternative hypothesis), but if the calculated probability is greater, then the null hypothesis is accepted. In case we reject H_0 , we run a risk of (at most the level of significance) committing an error of Type I, but if we accept H_0 , then we run some risk (the size of which cannot be specified as long as the H_0 happens to be vague rather than specific) of committing an error of Type II.

The above said steps of hypothesis testing can be shown using a flow chart



Hypothesis Testing of Means

Mean of the population can be tested presuming different situations such as the population may be normal or other than normal, it may be finite or infinite, sample size may be large or small, variance of the population may be known or unknown and the alternative hypothesis may be two-sided or one sided. The testing technique will differ in different situations. We may consider some of the important situations.

- 1. Population normal, population infinite, sample size may be large or small but variance of the population is known, Ha may be one-sided or two-sided:** In such a situation z -test is used for testing hypothesis of mean and the test statistic z is worked out as under:

$$Z = \frac{\bar{X} - \mu_{H_0}}{\sigma_p / \sqrt{n}}$$

Where, \bar{X} = Mean of the sample

μ_{H_0} = Hypothesised mean for the population

σ_p = Standard deviation of population

n = sample size

- 2. Population normal, population finite, sample size may be large or small but variance of the population is known, Ha may be one-sided or two-sided:** In such a situation z -test is used and the test statistic z is worked out as under (using finite population multiplier):

$$Z = \frac{\bar{X} - \mu_{H_0}}{(\sigma_p / \sqrt{n}) \times [\sqrt{(N-n)/(N-1)}]}$$

Where, \bar{X} = Mean of the sample

μ_{H_0} = Hypothesised mean for the population

σ_p = Standard deviation of population

n = sample size, N = size of population

- 3. Population normal, population infinite, sample size small and variance of the population unknown, Ha may be one-sided or two-sided:** In such a situation t -test is used and the test statistic t is worked out as under:

$$t = \frac{\bar{X} - \mu_{H_0}}{\sigma_s / \sqrt{n}} \quad \text{d.f.} = (n-1)$$

$$\text{and } \sigma_s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{(n-1)}}$$

Where \bar{X} = Mean of the sample
 μ_{H_0} = Hypothesised mean for the population
 σ_s = Standard deviation of sample
n = sample size

- 4. Population normal, population finite, sample size small and variance of the population unknown, and Ha may be one-sided or two-sided:** In such a situation t -test is used and the test statistic ' t ' is worked out as under (using finite population multiplier):

$$t = \frac{\bar{X} - \mu_{H_0}}{(\sigma_p/\sqrt{n}) \times [\sqrt{(N-n)/(N-1)}]} \quad \text{with d.f.} = (n-1)$$

$$\text{and } \sigma_s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{(n-1)}}$$

- 5. Population may not be normal but sample size is large, variance of the population may be known or unknown, and Ha may be one-sided or two-sided:** In such a situation we use z -test and work out the test statistic z as under:

$$Z = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}}$$

This applies in case of infinite population when variance of the population is known but when variance is not own, we use σ_s in place of σ_p in this formula
In case of finite population,

$$Z = \frac{\bar{X} - \mu_{H_0}}{(\sigma_p/\sqrt{n}) \times [\sqrt{(N-n)/(N-1)}]}$$

For example: A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance.

Solution: Taking the null hypothesis that the mean height of the population is equal to 67.39 inches, we can write:

$$\begin{aligned} H_0 &= \mu_{H_0} = 67.39'' \\ H_a &= \mu_{H_0} \neq 67.39'' \end{aligned}$$

and the given information as $X = 67.47''$, $\sigma p = 1.30''$, $n = 400$. Assuming the population to be normal, we can work out the test statistic z as under:

$$Z = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}}$$

$$= \frac{67.47 - 67.39}{1.30} / \sqrt{\frac{400}{n}}$$

$$= \frac{0.08}{0.065} = 1.231$$

As H_a is two-sided in the given question, we shall be applying a two-tailed test for determining the rejection regions at 5% level of significance which comes to as under, using normal curve area table: $R : |z| > 1.96$. The observed value of z is 1.231 which is in the acceptance region since $R : |z| > 1.96$ and thus H_0 is accepted. We may conclude that the given sample (with mean height = 67.47") can be regarded to have been taken from a population with mean height 67.39" and standard deviation 1.30" at 5% level of significance.

Hypothesis Testing for Difference Between Means

In many decision-situations, we may be interested in knowing whether the parameters of two populations are alike or different. For instance, we may be interested in testing whether female workers earn less than male workers for the same job. Here the technique of hypothesis testing for differences between means is explained. The null hypothesis for testing of difference between means is generally stated as $H_0 : \mu_1 = \mu_2$, where μ_1 is population mean of one population and μ_2 is population mean of the second population, assuming both the populations to be normal populations. Alternative hypothesis may be of not equal to or less than or greater than type as stated earlier and accordingly we shall determine the acceptance or rejection regions for testing the hypotheses. There may be different situations when we are examining the significance of difference between two means, but the following may be taken as the usual situations:

- Population variances are known or the samples happen to be large samples:** In this situation we use z -test for difference in means and work out the test statistic z as

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_{p1}^2}{n_1} + \frac{\sigma_{p2}^2}{n_2}}}$$

In case σ_{p1} and σ_{p2} are not known, we use σ_{s1} and σ_{s2} respectively in their places and they are calculated as follows:

$$\sigma_{s1} = \sqrt{\frac{\sum (X_{1i} - \bar{X}_1)^2}{(n_1 - 1)}}$$

$$\sigma_{s2} = \sqrt{\frac{\sum (X_{2i} - \bar{X}_2)^2}{(n_2 - 1)}}$$

Where \bar{X}_1 = mean of sample one; n_1 = number of items in sample one

\bar{X}_2 = mean of sample two; n_2 = number of items in sample two

- 2. Samples happen to be large but presumed to have been drawn from the same population whose variance is known:** In this situation we use z test for difference in means and work out the test statistic z as under:

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma_p^2 (\frac{1}{n_1} + \frac{1}{n_2})}}$$

In case σ_p is not known, we use $\sigma_{s_{1.2}}$. (combined standard deviation of the two samples) in its place which is calculated as follows

$$\sigma_{s_{1.2}} = \sqrt{\frac{n_1(\sigma_{s_1}^2 + D_1^2) + n_2(\sigma_{s_2}^2 + D_2^2)}{n_1 + n_2}}$$

$$\begin{aligned} \text{where } \bar{D}_1 &= \bar{X}_1 - \bar{X}_{1.2} \\ \bar{D}_2 &= \bar{X}_2 - \bar{X}_{1.2} \\ \text{and } \bar{X}_{1.2} &= \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2} \end{aligned}$$

- 3. Samples happen to be small samples and population variances not known but assumed to be equal:** In this situation we use t -test for difference in means and work out the test statistic t as under:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sum(X_{1i} - \bar{X}_1)^2 + \sum(X_{2i} - \bar{X}_2)^2}{n_1 + n_2 - 2} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$

with d.f. = $n_1 + n_2 - 2$

For example: The mean produce of wheat of a sample of 100 fields is 200 lbs. per acre with a standard deviation of 10 lbs. Another sample of 150 fields gives the mean of 220 lbs. with a standard deviation of 12 lbs. Can the two samples be considered to have been taken from the same population whose standard deviation is 11 lbs? Use 5 per cent level of significance.

Solution: Taking the null hypothesis that the means of two populations do not differ, we can write

$$H_0 = \mu_1 = \mu_2$$

$$H_a = \mu_1 \neq \mu_2$$

and the given information as $n_1 = 100$; $n_2 = 150$

$$\bar{X}_1 = 200 \text{ lbs.}; \bar{X}_2 = 220 \text{ lbs.};$$

$$\sigma_{s_1} = 10 \text{ lbs.}; \sigma_{s_2} = 12 \text{ lbs.}; \text{ and } \sigma_p = 11 \text{ lbs.}$$

Assuming the population to be normal, the test statistic z can be used as under:

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma_p^2 (\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$Z = \frac{200 - 220}{\sqrt{(11)^2 \left(\frac{1}{100} + \frac{1}{150}\right)}} = \frac{-20}{\sqrt{1.42}} = -14.08$$

As H_a is two-sided, we shall apply a two-tailed test for determining the rejection regions at 5 per cent level of significance which come to as under, using normal curve area table: $R : |z| > 1.96$. The observed value of z is -14.08 which falls in the rejection region and thus we reject H_0 and conclude that the two samples cannot be considered to have been taken at 5 per cent level of significance from the same population whose standard deviation is 11 lbs. This means that the difference between means of two samples is statistically significant and not due to sampling fluctuations.