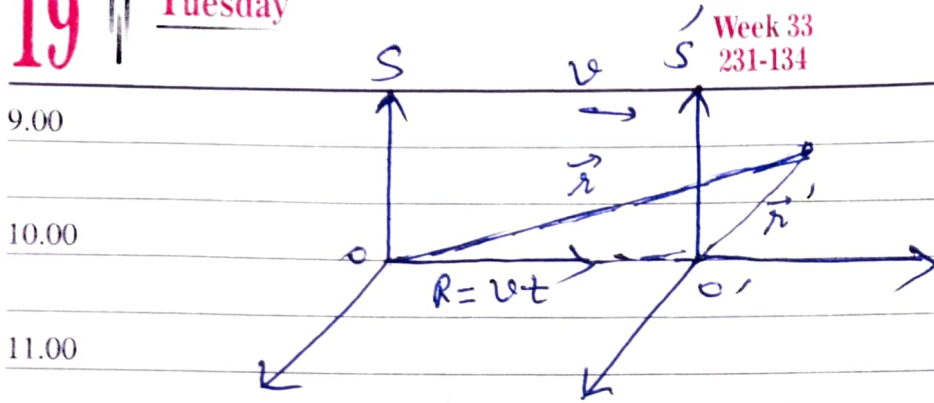


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Subject — Physics.

Title of the Topic — Inertial and Noninertial frames
of references.

Name of Program — B.Sc. (Physics Subsidiary)
Part - I
Paper - I (Group - B).



$$\therefore \boxed{\vec{r} = R + \vec{r}'} \Rightarrow \vec{r}' = \vec{r} - R$$

$$\Rightarrow \vec{r}' = \vec{r} - \vec{v}t$$

$$\Rightarrow \frac{d^2 \vec{r}}{dt^2} = \frac{d^2 \vec{r}'}{dt^2} \quad \left\{ \because \vec{v} = \text{constant} \right.$$

\Rightarrow acc^m of the particle is same in the two f.o.r.
 \Rightarrow if the particle be at rest in inertial frame S, it would also appear to be at rest in frame S'.

Here all the f.o.r moving with constant vel. w.r.t inertial frame are also inertial f.o.r.

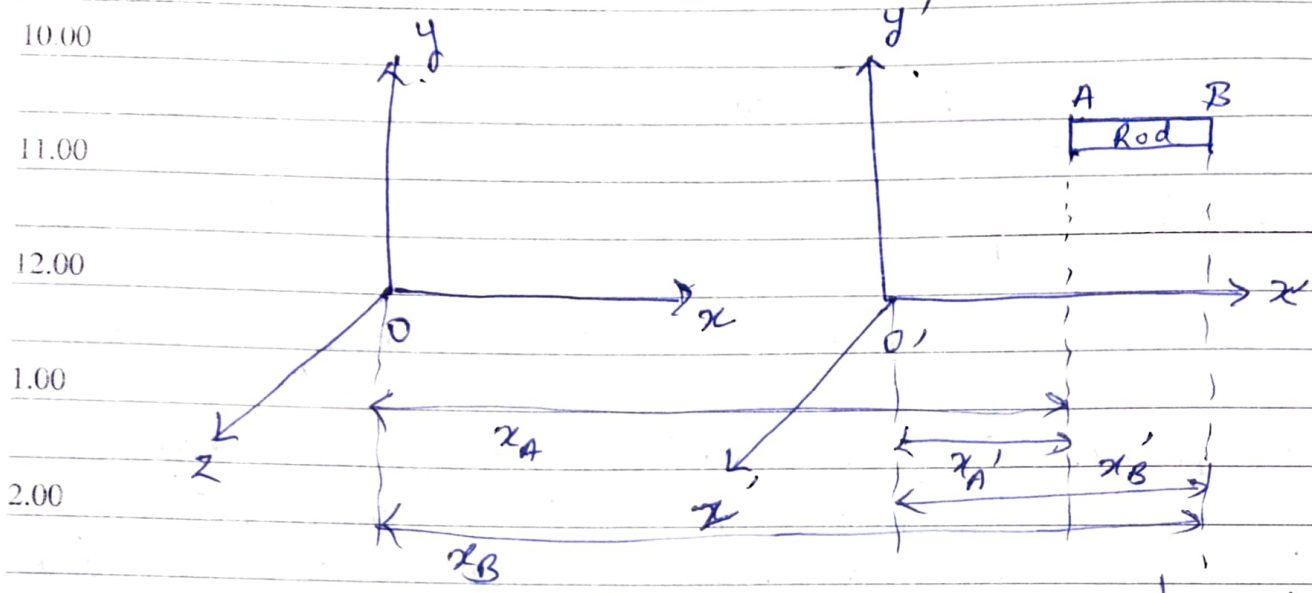
Non Inertial Frame of Ref.

A f.o.r. having an accelerated motion w.r.t. an inertial frame are called non-inertial f.o.r. Since a uniformly rotating frame has a centripetal acc^m. It is also an inertial frame of ref.

\therefore A non inertial frame is either a frame having uniform linear acc^m or a frame which is uniformly rotating.

Earth: - A non inertial f.o.r, because apart from going round the sun, it also spins about its own axis, This imparts to a body in the f.o.r. located on earth, a centripetal acc^m
 $= \omega^2 R = \left(\frac{2\pi}{24 \times 60 \times 60} \right)^2 \times 6.4 \times 10^8 = 3.4 \text{ cm/s}^2$, which is

9.00 (b) Transformation of distance of length.



S frame. $\therefore L = x_B - x_A$
 S' " $\therefore L' = x'_B - x'_A$

$x' = x - vt$
 $y' = y$
 $z' = z$
 $t' = t$

$L' = x'_B - x'_A$
 $L' = x_B - vt - (x_A - vt)$
 $L' = x_B - x_A + 0$
 $L' = L$

\therefore length is invariant under Galilean Transformation

(c) Transformation of vel.

Let u_x & u'_x be the vel. of the particle in S & S' respectively along x-dir.

$u_x = \frac{dx}{dt}$, $u'_x = \frac{dx'}{dt}$
 $u_y = \frac{dy}{dt}$, $u'_y = \frac{dy'}{dt} = \frac{dy}{dt} = u_y$
 $u_z = \frac{dz}{dt}$, $u'_z = \frac{dz'}{dt} = \frac{dz}{dt} = u_z$

(ii) Law of Conservation of mom. —

Consider 2 particles of masses m_1 & m_2 in frames, moving with vel \vec{u}_1 & \vec{u}_2 before collision, and with \vec{v}_1 and \vec{v}_2 after the collision.

From law of conservation of mom.

In S : $m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2$ — (1)

In S' : $m_1 \vec{u}'_1 + m_2 \vec{u}'_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$ — (2)

assume masses remain unaltered in 2 inertial frames

since

$$\left. \begin{aligned} \vec{u}_1 &= \vec{u}'_1 + \vec{v} \\ \vec{u}_2 &= \vec{u}'_2 + \vec{v} \\ \vec{v}_1 &= \vec{v}'_1 + \vec{v} \\ \text{and } \vec{v}_2 &= \vec{v}'_2 + \vec{v} \end{aligned} \right\} \text{--- (3)}$$

Putting (3) in (1)

$$m_1 (\vec{u}'_1 + \vec{v}) + m_2 (\vec{u}'_2 + \vec{v}) = m_1 (\vec{v}'_1 + \vec{v}) + m_2 (\vec{v}'_2 + \vec{v})$$

$$\Rightarrow m_1 \vec{u}'_1 + m_2 \vec{u}'_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

Law of conservation of mom is invariant to Galilean transf. i.e. it holds good in all inertial frames of ref. Thus whereas momentum, by itself is not invariant to Galilean transformation, the principle of conservation of mom. is.

(iii) The law of Conservation of Energy :-

According to conservation of Energy -

In S frame

$$\frac{1}{2} m_1 \vec{u}_1^2 + \frac{1}{2} m_2 \vec{u}_2^2 = \frac{1}{2} m_1 \vec{v}_1^2 + \frac{1}{2} m_2 \vec{v}_2^2 + E \text{ --- (1)}$$

$\checkmark E \rightarrow$ It is part of the energy of particles (before collision) appearing in some other forms like heat etc.

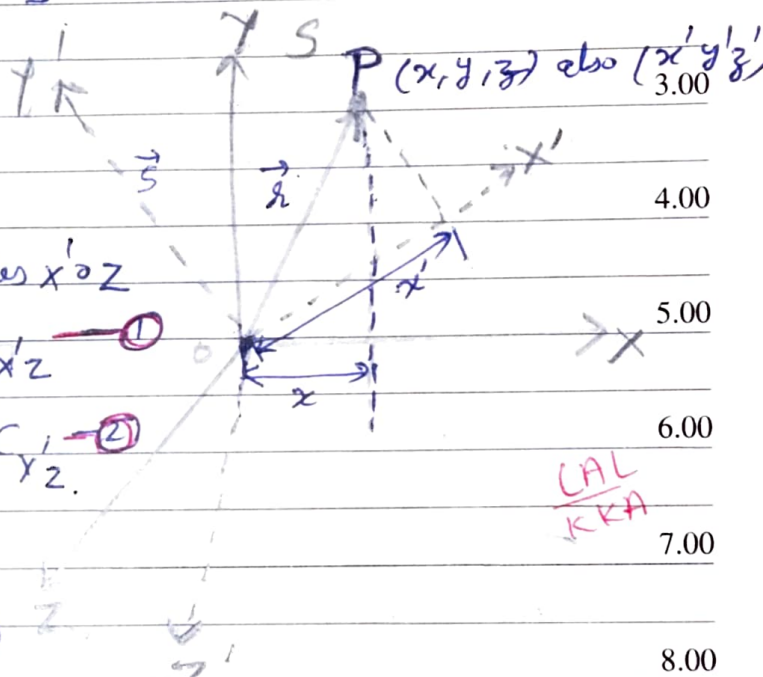
$$\Rightarrow m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{u}_1 + m_2 \vec{u}_2$$

which is law of conservation of linear momentum.

Transformation Eqn. for a frame of reference inclined to an inertial frame.

Let $S \rightarrow$ Inertial or a Galilean f.o.r and S' is f.o.r with its origin coinciding with that of S , but with its coord. axes inclined to that of S

$x' =$ Sum of components of x, y, z along the axis Ox' .



$$= x \cos x'Ox + y \cos x'Oy + z \cos x'Oz$$

$$= x C_{xx'} + y C_{x'y} + z C_{x'z}$$

Similarly $y' = x C_{y'x} + y C_{y'y} + z C_{y'z}$ (2)

and $z' = x C_{z'x} + y C_{z'y} + z C_{z'z}$ (3)

$$\therefore \frac{dx'}{dt} = \frac{dx}{dt} C_{xx'} + \frac{dy}{dt} C_{x'y} + \frac{dz}{dt} C_{x'z}$$

$$\frac{dy'}{dt} = \frac{dx}{dt} C_{y'x} + \frac{dy}{dt} C_{y'y} + \frac{dz}{dt} C_{y'z}$$

$$\frac{dz'}{dt} = \frac{dx}{dt} C_{z'x} + \frac{dy}{dt} C_{z'y} + \frac{dz}{dt} C_{z'z}$$

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KKA

9.00 $\therefore \frac{d^2 x'}{dt^2} = \frac{d^2 x}{dt^2} C_{x'x} + \frac{d^2 y}{dt^2} C_{x'y} + \frac{d^2 z}{dt^2} C_{x'z}$

10.00 $\frac{d^2 y'}{dt^2} = \frac{d^2 x}{dt^2} C_{y'x} + \frac{d^2 y}{dt^2} C_{y'y} + \frac{d^2 z}{dt^2} C_{y'z}$

11.00 $\frac{d^2 z'}{dt^2} = \frac{d^2 x}{dt^2} C_{z'x} + \frac{d^2 y}{dt^2} C_{z'y} + \frac{d^2 z}{dt^2} C_{z'z}$

1.00 Since S is an inertial frame, let particle does not
2.00 experience any force in S-frame $\therefore a = 0$

$\Rightarrow \frac{d^2 x}{dt^2} = \frac{d^2 y}{dt^2} = \frac{d^2 z}{dt^2} = 0$

3.00 \therefore we get $\frac{d^2 x'}{dt^2} = \frac{d^2 y'}{dt^2} = \frac{d^2 z'}{dt^2} = 0$

4.00 $\Rightarrow S'$ frame is also an inertial f.o.s.

5.00 \therefore Transformation Eqn. (1) (2) & (3) are Galilean Transf
6.00 Eqn. from S (inertial f.o.s.) to S' (inertial f.o.s.)

7.00 Inverse Galilean Transf. Eqn. from S' to S are

8.00 $x = x' C_{xx'} + y' C_{xy'} + z' C_{xz'}$

9.00 $y = x' C_{yx'} + y' C_{yy'} + z' C_{yz'}$

10.00 $z = x' C_{zx'} + y' C_{zy'} + z' C_{zz'}$

Transformation - Eqn. for a Rotating frame of Ref.

Consider an inertial frame of ref. S and another ref frame S' whose origin and coord. axes coincide with those of S initially. Now let S' starts rotating with a uniform ang. vel ω about common axis of z . \therefore after time t axis ox' & oy' of S' have turned through ωt w.r.t. ox & oy .

From last section

$$x' = x C_{xx'} + y C_{yx'} + z C_{zx'}$$

$$y' = x C_{y'x} + y C_{y'y} + z C_{y'z}$$

$$z' = z$$

$$C_{y'x} = \cos \gamma_{o'x} = \cos \left(\frac{\pi}{2} + \omega t \right) = -\sin \omega t$$

$$C_{y'y} = \cos \gamma_{o'y} = \cos \omega t$$

$$C_{y'z} = \cos \gamma_{o'z} = \cos \frac{\pi}{2} = 0$$

$$C_{xx'} = \cos \omega t$$

$$C_{yx'} = \cos \gamma_{ox'} = \cos \left(\frac{\pi}{2} - \omega t \right) = \sin \omega t$$

$$C_{zx'} = \cos \gamma_{oz'} = \cos \frac{\pi}{2} = 0$$

$$\therefore x' = x \cos \omega t + y \sin \omega t \quad \text{--- ①}$$

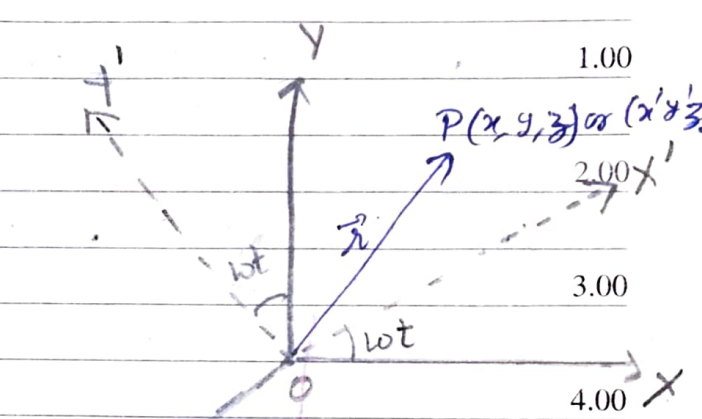
$$y' = -x \sin \omega t + y \cos \omega t \quad \text{--- ②}$$

$$z' = z \quad \text{--- ③}$$

$$\begin{aligned} x &= x' \cos \omega t - y' \sin \omega t \\ y &= x' \sin \omega t + y' \cos \omega t \\ z &= z' \end{aligned}$$

Relations ① \rightarrow ③ are called in frame of ref. S' rotating with a uniform ang. vel ω w.r.t inertial frame S .

Inverse Trans. Eqn.



9.00 $\therefore \frac{dx'}{dt} = -wx \sin wt + wy \cos wt + \frac{dx}{dt} \cos wt + \frac{dy}{dt} \sin wt$

10.00 $\Rightarrow \frac{dx'}{dt} = wy' + \frac{dx}{dt} \cos wt + \frac{dy}{dt} \sin wt$ — (4)

11.00 $\Rightarrow \frac{dx'}{dt^2} = w \frac{dy'}{dt} + \frac{dx}{dt} (-w \sin wt) + \frac{d^2x}{dt^2} \cos wt + \frac{dy}{dt} (w \cos wt)$
 $+ \frac{d^2y}{dt^2} (\sin wt)$

1.00 $\Rightarrow \frac{d^2x'}{dt^2} = w \frac{dy'}{dt} + w \left[-\frac{dx}{dt} \sin wt + \frac{dy}{dt} \cos wt \right] + \frac{d^2x}{dt^2} \cos wt + \frac{d^2y}{dt^2} \sin wt$

2.00 Similarly $\frac{dy'}{dt} = -wx \cos wt - wy \sin wt - \frac{dx}{dt} \sin wt + \frac{dy}{dt} \cos wt$

3.00 $\frac{dy'}{dt} = -wx' - \frac{dx}{dt} \sin wt + \frac{dy}{dt} \cos wt$ — (5)

4.00 $\frac{dz'}{dt} = \frac{dz}{dt}$ — (6)

5.00 Now diff. again (4) (5) & (6) w.r.t t —

6.00 $\frac{d^2x'}{dt^2} = w \frac{dy'}{dt} + w \left[-\frac{dx}{dt} \sin wt + \frac{dy}{dt} \cos wt \right] + \frac{d^2x}{dt^2} \cos wt + \frac{d^2y}{dt^2} \sin wt$

7.00 $= w \frac{dy'}{dt} + w \left[\frac{dy'}{dt} + wx' \right] + \frac{d^2x}{dt^2} \cos wt + \frac{d^2y}{dt^2} \sin wt$

8.00 $\frac{d^2y'}{dt^2} = -w dx' + \left[-w \frac{dx}{dt} \cos wt - w \frac{dy}{dt} \sin wt \right] - \frac{d^2x}{dt^2} \sin wt + \frac{d^2y}{dt^2} \cos wt$

9.00 $= -w dx' - w \left(\frac{dx'}{dt} - wy' \right) + \frac{d^2y}{dt^2} \cos wt - \frac{d^2x}{dt^2} \sin wt$

14 Sunday

9.00

$$d \frac{dz'^2}{dt^2} = \frac{dz^2}{dt^2}$$

Now Suppose P is at rest in frame S $\therefore acc^m = 0$

$$\Rightarrow \frac{dx^2}{dt^2} = \frac{dy^2}{dt^2} = \frac{dz^2}{dt^2} = 0$$

11.00

$$\therefore \frac{dx'}{dt^2} = 2\omega \frac{dy'}{dt} + \omega^2 x'$$

12.00

$$\frac{dy'}{dt^2} = -2\omega \frac{dx'}{dt} + \omega^2 y'$$

1.00

$$d \frac{dz'}{dt^2} = 0$$

2.00

\Rightarrow a force seems to be acting on it in frame S' producing an acc^m in it. \therefore S' is ~~not~~ a non inertial f.o.r.

3.00

\therefore NOTE Therefore (i) f.o.r in accelerated translation motion w.r.t. an inertial f.o.r.

5.00

(ii) a reference frame in uniform rotation about an inertial frame of ref. are both non-inertial f.o.r.

6.00

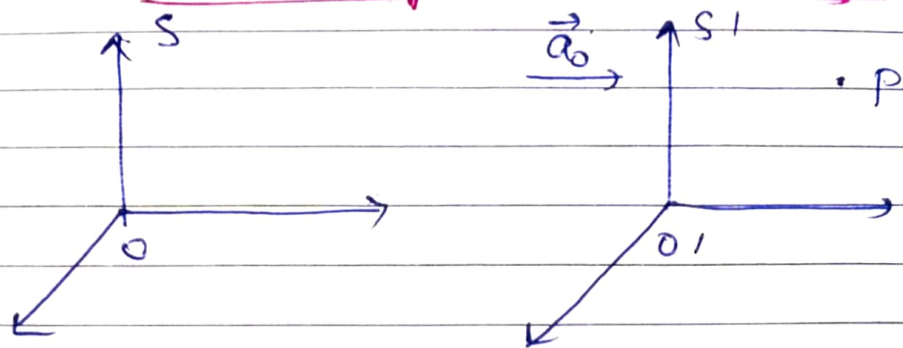
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The forces which acts on the particles in these f.o.r. are called fictitious forces.

8.00

9.00

Non-Inertial frames — Fictitious forces.



S' is moving with accⁿ a_0 w.r.t S .

$$\vec{a}' = \vec{a} - \vec{a}_0$$

$$\Rightarrow \boxed{\vec{F}_N = \vec{F}_I + \vec{F}_0}$$

$$\vec{F}_0 = -m\vec{a}_0$$

\vec{F}_N = force in Non-Inertial
 \vec{F}_I = " " Inertial
 \vec{F}_0 = fictitious force
 (∵ it is not an actual force)

Ex: of fictitious forces

① Lift is moving downward with accⁿ a_0 . Containing a ball of mass m .

∴ force on ball in inertial f.o.r = $\vec{F}_I = (mg)$ {∵ $a_0 = g$ }

& fictitious force on ball = $-m(a_0) = -m(g) = \vec{F}_0 = mg$

∴ force in Non-inertial f.o.r = $\vec{F}_I + \vec{F}_0$
 $= mg + mg = 0$

ie. particle is weightless and thus remains suspended in the air. It is same as condition obtained in artificial satellite.

NOTE! ① If 2 frames be in uniform motion (zero accⁿ) w.r.t each other, they are inertial f.o.r and it is impossible to find out which one is at rest & which one in motion.

② If however one of them be accelerated w.r.t other

9.00
10.00

fictitious force comes into play inside the accelerated frame. These forces are different from true or real forces.

4) If our frame is inertial frame, fictitious force is 0 i.e. it goes on diminishing rapidly with distance and almost zero at large distance.

5) If our frame is accelerated frame i.e. non-inertial then the real forces on particles or bodies at large distance from other bodies are negligible but fictitious force goes on increasing.

Prob: Calculate the fictitious force and observed force on a body of mass 5 kg in a frame of reference moving (i) vertically upward (ii) vertically downward with $a = 4 \text{ m/s}^2$

Let earth \rightarrow inertial f.o.r. & upward dirⁿ \rightarrow +ve.

Case (I) True weight of the body $F_I = mg$ (-ve)
 $= 5(-9.8) = -49 \text{ N}$
 $\Rightarrow 49 \text{ N}$ in downward dirⁿ

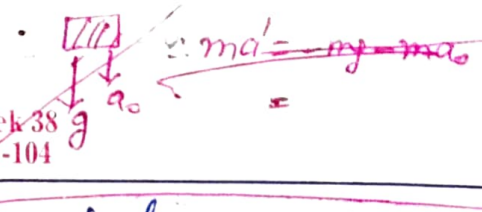
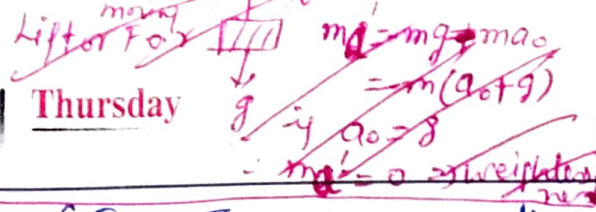
fictitious force acting on body $= F_0 = -ma_0$
 $= -5(4) = -20 \text{ N}$

$\Rightarrow 20 \text{ N}$ in downward dirⁿ.

Total force observed on body $F_N = F_I + F_0$
 $= -49 - 20 \text{ N}$
 $= -69 \text{ N}$

$\Rightarrow 69.0 \text{ N}$ in downward dirⁿ.

\Rightarrow Weight of the body becomes feels greater than the original weight



9.00 Case (II) True force acting on body = mg
 $\rightarrow = m(-9.8)$
 $F_I = -49 \text{ N}$
 10.00
 fictitious force " " " $F_0 = -ma$
 $= -5(-4)$
 $= 20 \text{ N}$
 11.00
 \therefore Total observed force on body = $-49 + 20$
 $\vec{F} = -29 \text{ N}$
 $\Rightarrow 29 \text{ N downwards}$

3.00 Prob 1 A frame of ref. is moving with an accⁿ of 5 m/s^2 downward. Find the apparent force & total force acting on a body of mass 10 kg falling freely relative to frame.

5.00 Sol As the body is falling freely, downward force on it in the inertial frame of the earth
 $\vec{F}_I = 0$
 $\therefore \vec{F}_N = +\vec{F}_0 \quad \forall \quad \vec{F}_0 = -ma_0$
 $= -10(-5)$
 $= +50 \text{ N}$

8.00 $\therefore 50 \text{ N upwards is the Total force acting on body}$

| | |
|---|---|
| <p>ie body becomes heavier than original weight</p> | <p>\therefore body feels lighter than the original weight</p> <p>Case: if $a_0 = g \therefore ma' = 0 \Rightarrow$ Condition weightless</p> |
|---|---|