Magadh Mahila College
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M A Economics
Semester II

Paper: Statistical Methods (CC 09)
Topic: Probability (Part iii) (Module 4)
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Probability

Probability is an expression of likelihood or chance of occurrence of an event. It ranges from 0 to 1—zero for an event which cannot occur and 1 for an event certain to occur.

Classical Definition (a priori probability)

- oldest and simplest approach
- assumes that outcomes of a random experiment (events) are equally likely
- probability of occurrence of event A is denoted by P (A) then

P(A) = no. of favourable cases/ Total no. of equally likely cases

Or, $p = P(A) = a/n$

$q = P(\text{not } A) = 1-p$

Where p= success of event q= failure or non- occurrence of event

n= total no. of equally likely cases

Also, $p + q = 1$

Emperical definition (Relative Frequency of Distribution)

- PA) is the limit of *a/n* as n tends to infinity
- the probability itself is the limit of the relative frequency as the number of observations increases indefinitely
-also called *a posteriori* probability
-derived from past experience and used in many practical problems like insurance, mortality tables etc.

There is also a **subjective approach** and an **axiomatic approach**.The first is based on probabilities assigned by an individual on basis of available data. The second follows certain axioms / postulates on which calculations are based.

Addition Theorem

If two events A & B are mutually exclusive (P (AB) = 0), the probability of the occurrence of A or B is the sum of individual probabilities of A and B.

$$P(A \text{ or } B) = P(A) + P(B)$$

For 3 or more cases

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

If events are not exclusive the theorem is modified as

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \text{ or } P(A\cap B) = P(A) + P(B) - P(A\cup B)$$

For 3 or more events,

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A\cup B) - P(A\cup C) - P(B\cup C) + P(A\cup B\cup C)$$

Multiplication Theorem

If 2 events are independent then the probability that they both will occur is given by the product of their individual probabilities

$P(A\&B) = P(A) \times P(B)$

$P(A, B \text{ and } C) = P(A) \times P(B) \times P(C)$

For Independent events A_1, A_2, A_3,……A_n with respective probabilities occurrences p_1, p_2, p_3…… p_n, the probability of the occurrence at least one of *n* events is

P (happening of at least 1event) =1 –p(happening of none of the events)

Conditional Probability

A & B are dependent events if B can only occur if A is known to have occurred or vice versa. The probability attached with such an event is called conditional probability and is given by:

$P(B/A) = P(AB) / P(A)$ $\qquad\qquad$ $P(A/B)= P(AB)/P(B)$

Multiplication theorem is then modified as

$P(AB) = P(B)\ P(A/B)$ $\qquad\qquad$ $P(AB)= P(A) \times P(B/A)$

For 3 events,

$P(ABC) = P(A) \times P(B/A) \times P(C/AB)$

Bayes theorem

- named after British mathematician Rev. Thomas Bayes (1702-61) published 1763.

-probability of some event A, given that of another event B is has been on will be observed is P(A/B) which can be calculated using the theorem

$$P(A/B)= P(B/A)\ P(A) / \textstyle\sum_{i=1}^{k} P(B/A)\ P(A)$$

-Probability of event A_1 given B = $P(A_1/B)$ = $P(A_1\&B)/ P(B)$

-Prob. of event A_2 given B is = $P(A_2/B) = P(A_2\&B)/ P(B)$

And $\qquad$ $P(B) = P(A_1\&B) + P(A_2\&B)$;

$P(A_1\&B) = P(A_1) \times P(B/A_1)$; $\qquad$ $P(A_2\&B)= P(A_2) \times P(B/A_2)$

In general, let A_1, A_2 ,….. A_i ….A_n be a set of *n* mutually exclusive & collectively exclusive events. If B is another event and P(B)= 0 then

$$P(A_1/B) = P(B/A_1)\ P(A_1) / \textstyle\sum_{i=1}^{k} P(B/A1)\ P(A1)$$

Mathematical Expectation

X is discrete random variable with values X_1, X_2 ... X_k with probabilities, p_1 …..p_k where $p_1 + p_2 + \ldots\ldots + p_k = 1$ then $E(X) = p_1X_1 + p_2X_2 + \ldots\ldots\ldots + p_kX_k$

Or, expected values equals the sum of each particular value within the set X multiplied by the probability that X equals that value

Questions for practice

1 From a pack of 52 cards, one card is drawn at random. What is the probability that it will be
i)a king ii) an ace of spade iii) a card of black colour

Ans- i) 1/13, ii)1/52 iii) 1/2

2. Find the probability of obtaining a total of 2 or 8 or 12 on a throw of two dice. Ans - 7/16

3. Out of 3 events, X, Y and Z, only one can happen at a time. Odds against X are 5:3, against Y are 4:2. Find the odds against the happening of Z. Ans – 17: 7

4. A committee of 5 is to be formed out of a group of 8 boys and 7 girls. Find the probability the committee has 3 boys & 2 girls. Ans- 1176/3003

5. 4 cards are drawn successively at random without replacement. What is the probability that all the four will be aces? Ans- 1/270725

6. A problem i given to three students A, B and C. Their chances of solving it are ½, 1/3 and ¼ respectively. What is the probability that the problem will be solved? Ans- 3/4