Lecture Notes of B.Sc.(HONS.) PHYSICS ,Part-II, Paper -IV

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Topic: Betatron

It is a device which is used to accelerate electrons to high energies .As fast moving electrons are called beta particles so that the device is called "Betatron" it can accelerate electron upto >=300 Mev.

Principle : Betatron condition

In Betatron the electrons are put in a nonuniform magnetic field therefore electron starts moving in a circular orbit in a plane perpendicular to the direction of applied magnetic field .

These electron are continuously accelerated due to the emf. Produced by an increasing flux through the orbit . Let us consider an electron moving in a circular orbit . The magnetic field is non uniform in space and increasing with time . Suppose at a given instant B_1 is the any flux through the orbit and Br is the flux density at the orbit itself . The instantaneous magnetic flux φ through the orbit is given by –

φ = avg. flux density through the orbit X area enclosed by orbit

$\varphi = B_1 \times \pi r^2 = B_1 \pi r^2$

As the magnetic field is increasing with time, the magnetic flux φ through the orbit is increasing at the rate $\frac{d\varphi}{dt}$. As a result an emf.is induced in the orbit;

Induced emf. = $\frac{d\varphi}{dt}$ = V

Thus the Work done on electron in one complete revolution is given by;

W= e x emf.

= e V

Therefore tangential force acting on electron is given by;

$$F = \frac{work\ done\ in\ one\ revolution}{distance\ moved\ in\ one\ revolution}$$

$$F = \frac{w}{2\pi r} = \frac{eV}{2\pi r} = \frac{eV}{2\pi r} = \frac{e\frac{d\varphi}{dt}}{2\pi r}$$

$$F = \frac{e\frac{d(\pi r r\, B1)}{dt}}{2\pi r} = \frac{er\frac{dB1}{dt}}{2} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The electron is accelerated by this force acting upon it, which is arising due to emf produced by increasing average flux density B_1 through the entire orbital area. As B_1 increases the electron energy increases and tends to move in an orbit of increasing radius . Now electron the electron is moving in a circular orbit of radius r in a magnetic field perpendicular to it .The magnetic flux density at the orbit is B_r. If v is the instantaneous speed of the electron . The in instantaneous radial force =B_r e v = centripetal force

$\Rightarrow$ B_r e v = $\frac{m\, vv}{r}$

⇨ Instantaneous momentum of electron = p = mv = B_r e v

Now by Newtons 2nd Law of motion F = $\frac{dp}{dt}$

⇨ F = $\frac{d(\text{Br e v})}{dt} = er\frac{dBr}{dt}$.....(2)

As Br increases, the electron tends to move in an orbit of decreasing radius.Since the force given by eqn. (1) and (2) have opposite actions on the radius of the orbit ,the former tends to increase while the later tends to decrease the radius .Hence to preserve constant radius of the orbit ,these two forces must be equal.

i.e. $\frac{er\frac{dB1}{dt}}{2} = er\frac{dBr}{dt}$

Integrating both sides of eqn. assuming that $B_1 = B_r = 0$ at t =0

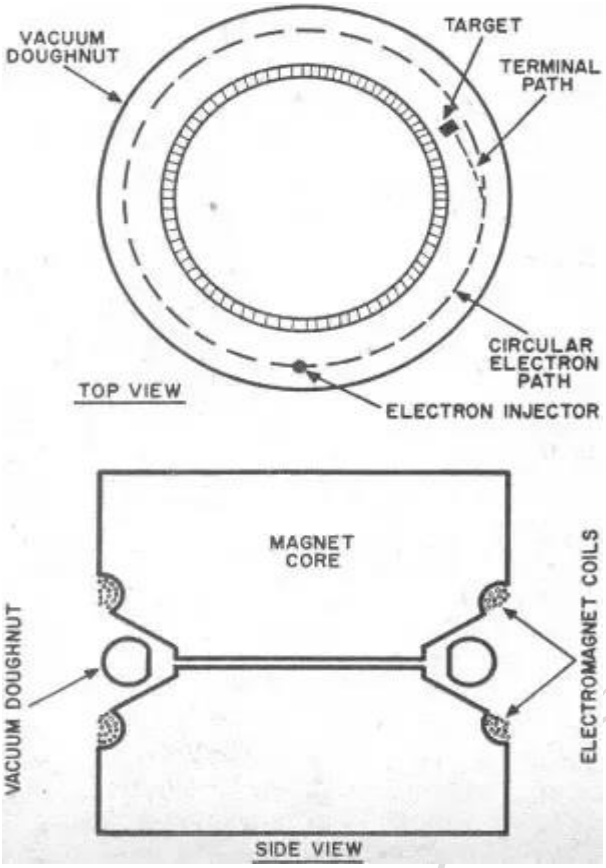
Therefore $\frac{erB1}{2} = erB_r$

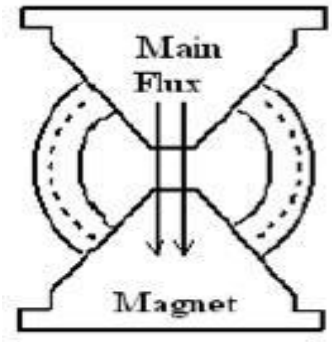
⇨ $\boxed{B_1 = 2\,B_r}$ ..... **Betatron Condition**

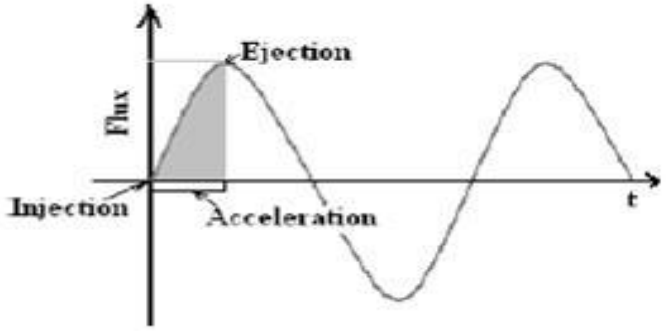
Thus the average magnetic flux density through the orbit must always be twice the flux density of the orbit. This is operating condition of Betatron. This condition is achieved by suitably designing the size and the shape of the magnet providing the magnetic field. When this is done , the electrons once put in the orbit ,go again and again around the same orbit with continuously increasing speed so long as the magnetic field is increasing (with time).

In Betatron the increase in speed around the circular orbit is continuous and there is no synchronism to be maintain therefore any relativistic increasing in mass of the electron and the slowing down of the rate of revolution don't affect the operation of the Betatron .

Construction : The construction of Betatron is show in figure. It consist of a large highly evaluated annular tube DD called a "doughnut" tube in which the electron revolve. Their orbit is a circle at right angle to the plane of the page. They are shown entering the page at the right (.) and emerging the page at the left (X). The tube DD is placed between the specially designed pole pieces of a large electromagnet energized by an alternating current. The magnet satisfied the Betatron condition by providing a large flux density in the central part of the field . Since the magnet is energized by alternative current, and increasing magnetic field in a given direction in obtained only during one quarter cycle in which the current increases from zero to the peak value . Hence the electrons are kept in their orbit only during this part of each cycle, and the output electron beam is in the form of pulses .







Working : The electrons to be accelerated are injected in to the doughnut tube DD by means of electron gun . The electrons are injected at the start of a quarter cycle in which the magnetic field increases in a given direction from zero to the peak value. These are soon captured in to a stable orbit and makes several hundred thousand revolution during the quarter cycle, being accelerated continuously. At the end of each accelerating quarter cycle the orbit is momentarily expanded by means of an auxilliary magnetic field (produced by sending a pulse of an extra current through auxilliary coil) So that the electron strikes the target which produces Gamma rays or X-rays or

emerges through a window . Betatron can produce electrons with energy greater than 300 Mev .

Applications of Betatron :-

1. It provides high energy beam of electron approximately 300 Mev.
2. It can be used as a source of X-rays and Gamma rays if electron beam is made to direct on a metal plate.
3. The X-rays which are produced with the help of Betatron can be used in industrial and Medical fields .
4. High energy electrons can be used in particle Physics.