DIFFRACTION DUE TO A PLANE DIFFRACTION GRATING

Bp, Qsnpgw_ P_lg
Escqr D_asjrw
Bcn_rr k clr md Nfwqgaq
K_e_bf K_fgj_ Amjjcec* NS
Ck_gj gb+ qsnpgw_,nfwqgaq>ek_gj,amk

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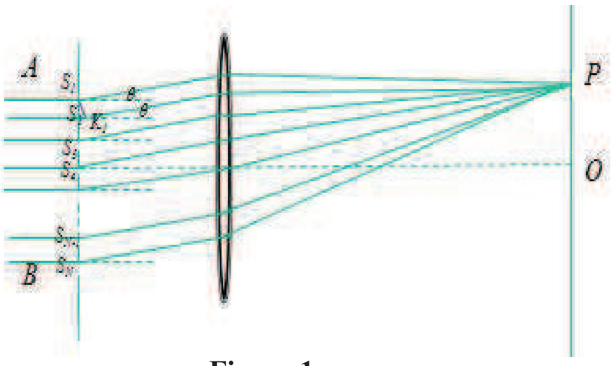

Figure 1

Here, S_1, S_2, S_3….. $S_N$are N narrow slits, in between points A and B. Let b= width of slit, d= width of opaque part between two slits.

The amplitude from each slit in the direction θ is

$$R_0 = \frac{A\sin\alpha}{\alpha}$$

Where $\alpha = \frac{\pi b}{\lambda}\sin\theta$ (As , in case of Single slit Fraunhofer diffraction)

The path difference between the wavelets from S_1 and S_2 in the direction θ is

$$S_2K_1 = (b+d)\sin\theta$$

Hence the phase difference between them

$$\frac{2\pi}{\lambda}(b+d)\sin\theta = 2\beta \text{, say}$$

If N be the total number of slits in the grating, the resultant amplitude in the direction of θ will be

$$R = R_0 \frac{\sin N\beta}{\sin\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\sin\beta} \qquad \text{....... (1)}$$

Thus, the resultant intensity at point P is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad \text{....... (2)}$$

The factor $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ gives the intensity distribution due to single slit, while $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the distribution of intensity in the diffraction pattern due to the interference in the waves due to N slits.

Principal Maxima

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The intensity will be maximum when

$$\sin\beta = 0 \Rightarrow \beta = \pm n\pi$$

Where, $n = 0,1,2,3.....$

This result in

$$\frac{\sin N\beta}{\sin\beta} = \frac{0}{0} \text{ (Indeterminate)}$$

Applying L' Hospital rule

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$\lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta} \Rightarrow \pm N$$

This result in

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 N^2$$

The condition for principal maxima is

$$\sin\beta = 0 \qquad \text{or } \beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(b+d)\,Sin\,\theta = \pm n\pi$$

$$(b+d)\,Sin\theta = \pm n\lambda \qquad \text{.......} \quad (3)$$

For n = 0, we get $\theta = 0$ and this gives the direction of zero order principal maxima. The value of n = 1, 2, 3 etc. gives the direction of first, second, third etc. order principal maxima.

Minima

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The intensity will be minimum when

$$\sin N\beta = 0 \text{ but } \sin\beta \neq 0$$

Therefore, $$N\beta = \pm m\pi \qquad \text{.......} \quad (4)$$

Missing Orders

As the resultant intensity due to N-parallel slits (plane diffraction grating) is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

Where, $$\alpha = \frac{\pi b}{\lambda}\sin\theta$$

And $$\beta = \frac{\pi}{\lambda}(b+d)\sin\theta$$

Now the direction of principal maxima in grating spectrum is given as

$$(b+d)\,Sin\theta = n\lambda \qquad \text{.......} \quad (5)$$

Further the direction of minima of a single slit pattern is

$$b\,Sin\theta = m\lambda \qquad \text{.......} \quad (6)$$

Where *m= 1, 2, 3……*

If both the conditions are simultaneously satisfied, a particular maximum of order n will be absent in the grating spectrum, these are known as absent spectra (or missing order spectrum).

Dividing equation (5) by equation (6), we get

$$\frac{b+d}{b} = \frac{n}{m} \qquad \text{....... (7)}$$

If b = d, then 2nd, 4th, 6th etc. orders maxima will be missing in the grating diffraction pattern.

If d = 2b, then 3rd, 6th, 9th etc. orders maxima will be missing in the grating diffraction pattern.

Maximum Number of Order Available in a Grating

The grating equation is $(b+d)\,Sin\theta = n\lambda$

or

$$n = \frac{(b+d)\,Sin\,\theta}{\lambda} \qquad \text{....... (8)}$$

Maximum possible value of θ is 90^0.

Therefore, Maximum possible order will be

$$n_{\max} = \frac{(b+d)\,Sin\ 90}{\lambda} = \frac{(b+d)}{\lambda} \qquad \text{....... (9)}$$