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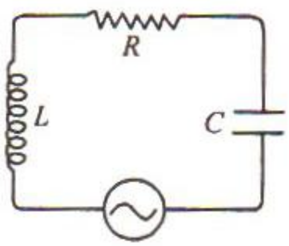
Name of Program –Physics (Hons), Part II

Paper –IV, Group B

LCR in Seris resonant circuit -

Consider an *A.C.* circuit having a resistance R, an inductance L and a capacitance C connected in *series* as shown in Fig. 11.6. Let an alternating *e.m.f.* $E = E_0 \sin \omega t$ be applied to the circuit, so that at any instant, q is the charge on the capacitor, i is the current in the circuit and $\frac{di}{dt}$ the rate at which the current varies. The applied *e.m.f.* E must produce a potential difference Ri across the resistance, a potential difference $\frac{q}{C}$ across the capacitor and should overcome a back *e.m.f.* $-L\frac{di}{dt}$ due to the inductance.



$$\therefore \quad E - L\frac{di}{dt} = Ri + \frac{q}{C}$$

or
$$E_0 \sin \omega t = L\frac{di}{dt} + Ri + \frac{q}{C} \qquad ...(i)$$

The current in the circuit will vary harmonically but will have a different amplitude and will differ in phase by an angle, say ϕ. Let the current in the circuit be represented by

$$i = I_0 \sin(\omega t - \phi)$$

then
$$\frac{di}{dt} = I_0 \omega \cos(\omega t - \phi)$$

and
$$q = \int i\, dt = \int I_0 \sin(\omega t - \phi)\, dt$$
$$= -\frac{I_0}{\omega}\cos(\omega t - \phi)$$

Substituting these values in (i), we have

$$E_0 \sin \omega t = L\omega I_0 (\cos \omega t - \phi) + RI_0 \sin(\omega t - \phi) - \frac{I_0}{C\omega}\cos(\omega t - \phi)$$

or
$$E_0 \sin \omega t = I_0\left[R\sin(\omega t - \phi) + \left(L\omega - \frac{1}{C\omega}\right)\cos(\omega t - \phi)\right] \qquad ...(ii)$$

To solve this equation put $R = a\cos\phi$

and
$$\left(L\omega - \frac{1}{C\omega}\right) = a\sin\phi$$

$\therefore \quad a = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$

and $\quad \tan\phi = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$

Substituting the values of R and $\left(L\omega - \frac{1}{C\omega}\right)$ in (*ii*), we have

$$E_0 \sin\omega t = I_0\, a\, [\cos\phi \sin(\omega t - \phi) + \sin\phi \cos(\omega t - \phi)]$$

$$= I_0\, a \sin(\omega t - \phi + \phi) = I_0\, a \sin\omega t$$

$$= I_0 \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \sin\omega t$$

$$\therefore \quad E_0 = I_0 \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

or $$I_0 = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$\therefore \quad I_v = \frac{E_v}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \qquad ...(iii)$$

The current at any instant is given by

$$i = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \sin(\omega t - \phi)$$

Phase angle. The phase angle

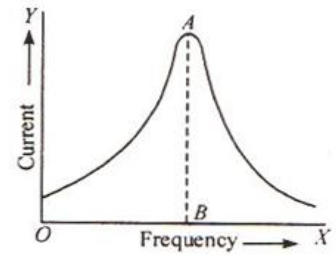
$$\phi = \tan^{-1} \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$$

Impedance. The quantity $\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$ is known as the effective resistance of the *A.C.* circuit containing resistance, inductance and capacitance. It is called *impedance* and is denoted by Z. *$L\omega$ is the reactance due to inductance* and is denoted by X_L and $\frac{1}{C\omega}$ *is the reactance due to capacitance.* It is denote by X_C.

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The *e.m.f.* and current have a phase difference of ϕ given by the relation

$$\phi = \tan^{-1} \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R} = \tan^{-1} \frac{X_L - X_C}{R}$$



Resonance. The variation of current with frequency in an electric circuit containing resistance, inductance and capacitance in series is shown in Fig. 11.8.

When the frequency of the applied e.m.f. equals the natural frequency of the electrical circuit the current reaches a maximum value and the circuit is said to be in resonance with applied e.m.f. This is given by AB.

Condition for sharp resonance. In an *A.C.* circuit containing a resistance R, inductance L, capacitance C to which an applied *e.m.f.* of value E_v having an angular frequency ω is applied, the current I_v is given by

$$I_v = \frac{E_v}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

For the current in the circuit to be maximum $R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2$ must have a minimum value.

This is so when

$$L\omega - \frac{1}{C\omega} = 0$$

or

$$L\omega = \frac{1}{C\omega}$$

or

$$\omega = \sqrt{\frac{1}{LC}}$$

$\therefore$

$$n = \frac{1}{2\pi\sqrt{LC}}$$

Hence *sharp resonance* takes place when the reactance due to capacitance $\frac{1}{C\omega}$ is equal to the reactance due to inductance $L\omega$. For resonance the frequency of the applied *e.m.f.* must be given by

$$n = \frac{1}{2\pi\sqrt{LC}}$$

i.e., the frequency of the applied *e.m.f.* must be equal to the natural frequency of the circuit.

Current At resonance $L\omega - \frac{1}{C\omega} = 0$

$\therefore$ Current

$$I_v = \frac{E_v}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} = \frac{E_v}{R}$$

This is the *maximum* value of current in the circuit.

Hence, at resonance, out of L, C and R it is the resistance R which controls the current.

Phase angle At resonance $L\omega - \frac{1}{C\omega} = 0$. As the phase angle ϕ between current and applied *e.m.f.* is given by

$$\phi = \tan^{-1} \frac{L\omega - \frac{1}{C\omega}}{R}, \quad \phi = 0$$

Thus at resonance the current is maximum and *e.m.f. and current are in phase*.

Quality factor. In a series *RLC* circuit at *resonance*

$$L\omega = \frac{1}{C\omega}$$

As the same current flows through each circuit element the drop of potential across the inductance is *equal* in magnitude but *opposite* in phase to the drop of potential across the capacitance.

The ratio of the potential drop across the inductance or capacitance to the potential drop across the resistance (*or the applied voltage*) *is called the Quality factor or Q of the circuit.*

If I is the current in the circuit, then

$$Q = \frac{\text{Potential drop across } L}{\text{Potential drop across } R} = \frac{I\,L\omega}{IR} = \frac{L\omega}{R}$$

As at resonance $L\omega = \frac{1}{C\omega}$, $\omega = \frac{1}{\sqrt{LC}}$

$$\therefore \quad Q = \frac{L\omega}{R} = \frac{L}{R}\cdot\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

In general, the ohmic resistance R in a series RLC circuit is *small* as compared to the inductive or the capacitative reactance. There is a very high potential drop across the inductance or capacitance as compared to the potential drop across the resistance. This potential drop at resonance can be much greater than the applied voltage.

Resonance. In a series LCR circuit, the current is given by

$$I_v = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

The current has a maximum value when the impedance of the circuit

$$\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

is a minimum. The value of the impedance is the least when

$$L\omega - \frac{1}{C\omega} = 0$$

or $$L\omega = \frac{1}{C\omega}$$

or $$\omega = \sqrt{\frac{1}{LC}}$$

or Frequency $$n = \frac{1}{2\pi\sqrt{LC}}$$

In other words, when the frequency of the applied *e.m.f.* is equal to the *natural frequency* of the LC circuit, the current is a maximum and *resonance* is said to take place. Such a circuit is known as *series resonant* circuit and the corresponding frequency is known as *resonant frequency*. In series resonance the voltage across the inductance is equal to the voltage across the capacitance but these have a phase difference of 180° between them.

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