Name of the teacher – Ms. Sonu Rani, Assistant Professor
Magadh Mahila College, P.U.

Subject – PHYSICS.

Title of the topic – Special theory of relativity.

Name of Prog. – B.Sc. (H.) PHYSICS.
Part-I, Paper-I (Group-A)

9.00

Special Theory of Relativity

10.00

Principle of Relativity — All the phenomenon in nature should appear the same from 11.00 all the frames of reference.

12.00

Principle of Special relativity / Galilean " — It is principle of relativity applicable b/w inertial frames 1.00

Principle of General Relativity — It is principle of relativity also applicable to noninertial 2.00 frames of ref.

3.00

Newton's laws of mechanics obey Galilean Relativity.
Maxwell's " " electromagnetism don't obey " " 4.00
Maxwell assumed that light moves in a medium called ether which fills all vaccum and is at rest everywhere. ∴ Ether was 5.00 absolute f.o.r. & his Eqn. are valid in f.o.r. being at rest w.r.t. ether or an absolute f.o.r. 6.00

Let $c \rightarrow$ vel. of light in ether
$v \rightarrow$ vel. of (Inertial frame) Earth w.r.t ether. 7.00
$c' \rightarrow$ vel. of light w.r.t Earth.

8.00

$$\therefore \boxed{c' = c - v}$$

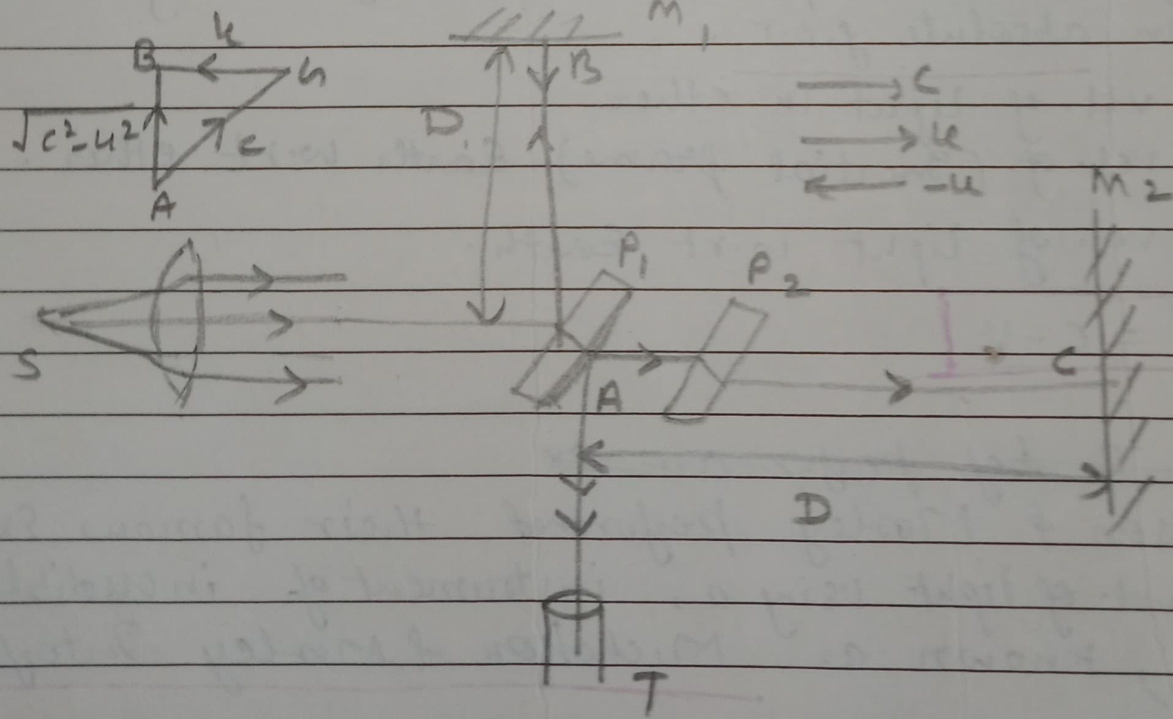
9.00

~~So we will bef perfor an ex~~

So Michelson & Morley performed their famous Expt. to measure vel. of light using an instrument of incredible sensitivity, known as Michelson & Morley Interferrome

Michelson and Morley Experiment

The apparatus used by them is known as interferometer since it depends upon the principle of interference of light. S is source of monochromatic light, a ∥ beam from which falls upon a thin ∥ sided glass plate P_1 thinly silvered on the back surface. The incident beam is thus split up into 2 parts at A at right angles to each other and of equal intensity, a reflected one, travelling at mirror M_1 at T along its own path and gets refracted through P_1 on to the telescope T. The latter similarly gets reflected back as it falls normally on Mirror M_2 at C and is then reflected downward from the back surface of P_1 along the same path as the former to enter the telescope. The two mirrors M_1 & M_2 are heavily silvered on the front face (to avoid multiple reflections) and are arranged at right angles to each other at same distance D from plate P_1.



From the fig. the beame reflected upwards to M_1 transverse the thickness of plate P_1 thrice whereas the one refracted on to mirror M_2 does so only once. To make their paths through glass and air equal, therefore a compensating plate P_2, identical with P_1 is arranged parallel to it.

If the beam of light be exactly ∥ and if distance b/w AB and AC from P_1 to the two mirrors respectively be the same D, the two parts of the beam from M_1 & M_2 arrive at the telescope in phase and the field of view appear bright. But since the beam of light incident on P_1 is ~~almost~~ always a slightly divergent one, ∴ some of the rays reaching the telescope are in phase and some are out of phase. So what we observe ~~on~~ through the telescope is an interference pattern of dark and bright fringes.

Let $u \rightarrow$ vel. of apparatus (i.e of earth) relative to ether from left → Right.

$c \rightarrow$ vel. of light in ether.

$c' \rightarrow$ vel. of light w.r.t apparatus (earth)

∴ From A → C $\quad c' = c - u$

∴ Time take to cover the distance D from A → C $= \dfrac{D}{c-u} = t_1$

from C → A $\quad c' = c + u \quad \{ u = -ve$ from Right → left

∴ Time taken to cover distance D from C → A $= \dfrac{D}{c+u} =$

∴ Time taken by light to travel from A → C & back to A

$t = t_1 + t_2$

$$t = \left(\frac{D}{c-u}\right) + \left(\frac{D}{c+u}\right)$$

$$\boxed{t = \frac{2cD}{c^2-u^2} = \frac{2D/c}{1-u^2/c^2}} \quad \text{—— (1)}$$

For the beam proceeding upwards from A→B must move along the dirⁿ· A→G so that the resultant of c & ether drift (−u) is along AB

This Resultant vel. $= \sqrt{c^2-u^2}$

∴ Time taken to cover the distance D from A→B

$$= t_1' = \frac{D}{\sqrt{c^2-u^2}}$$

& " " " " " distance D from B→A

$$t_2' = \frac{D}{\sqrt{c^2-u^2}} = t_1'$$

∴ total time taken by light to travel from A→B and back, $t' = t_1' + t_2' = 2t_1' = \frac{2D}{\sqrt{c^2-u^2}} = \frac{2D/c}{\sqrt{1-u^2/c^2}}$

Both t & t' have same numerator but in the denominator $\frac{u^2}{c^2} << 1$ ∴ $1-\frac{u^2}{c^2} < 0$ ∴ $\sqrt{1-\frac{u^2}{c^2}} > 1-\frac{u^2}{c^2}$

∴ t' is slightly less than t.

∴ Difference in time $\Delta t = t - t' = \frac{2D/c}{\left(1-\frac{u^2}{c^2}\right)} - \frac{2D/c}{\sqrt{1-\frac{u^2}{c^2}}}$

$$= \frac{2D}{c}\left(1-\frac{u^2}{c^2}\right)^{-1} - \frac{2D}{c}\left(1-\frac{u^2}{c^2}\right)^{-1/2}$$

Using Binomial thm —

$$\Delta t = \frac{2D}{c}\left[\left(1+\frac{u^2}{c^2}\right)-\left(1+\frac{1}{2}\frac{u^2}{c^2}\right)\right] = \frac{2D}{c}\left(\frac{u^2}{2c^2}\right)$$

$$= \frac{Du^2}{c^3}.$$

$\therefore$ distance covered by light in time Δt

$$= \Delta t \times c = \frac{Du^2}{c^3}c = \frac{Du^2}{c^2}.$$

$\Rightarrow$ optical path AC is longer by $\frac{Du^2}{c^2}$ ~~times~~ than the optical path AB.

$\therefore$ It's the path diff. b/w two parts of incident beam reflected from M_1 & M_2 due to motion of the apparatus (Earth.)

If the apparatus is now turned through 90° so that AB comes into the line of motion and AC $\perp$ to it, so that the optical path AB is longer than AC by same amount $\frac{Du^2}{c^2}$.

$\therefore$ Total Path diff. introduced b/w two beams $= \frac{2Du^2}{c^2}$

Due to this effect the interference pattern in the field of view of telescope will shift say through n fringes. $\Rightarrow$ path diff $= n\lambda$

$$\Rightarrow \quad n\lambda = \frac{2Du^2}{c^2}$$

$$\Rightarrow \quad n = \frac{2Du^2}{c^2\lambda}$$

Using
u = orbital vel of earth = vel. of apparatus = 30 km/s
D = 11 m
λ = 6×10^{-5} m

9.00
10.00
11.00
12.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00

$\Rightarrow$ $n = 0.37 \simeq 0.4 \Rightarrow$ No fringe shift was observed.

$\Rightarrow$ There is no relative motion b/w the earth & ether.

Speed of light

Various expt are performed to measure the speed of light over the years, It is independent of the place of measurements, freq., nature and motion of source & dirn of its propagation. It is also constant w.r.t all inertial frames of reference. Thus experiments help us to accept the principle

"The speed of light in free space is a universal const"

Special Theory of Relativity

The two basic postulates on which the theory rests may be stated as —

1.) The principle of relativity :— The laws of physics all takes the same identical form for all f.o.r. in uniform relative motion. i.e. for all the inertial frames of reference.

2) The principle of constancy of speed of light — The vel of light in free space is the same (c) relative to any inertial f.o.r. ie. it is invariant to transformation from one inertial frame to another and has thus same value 3×10^8 m/s for all observers irrespective of their state of motion.

NOTE — General theory of Relativity

It deals with the accelerated f.o.r. ie Non Inertial f.o.r.
Here we will deal with Special theory of Relativity.

Galilean & Lorentz Transformation.

Suppose a flash of light be emitted from the origin when point O and O' of the two frames of ref. S & S' just cross each other. Each observer thinks himself to be at the centre of a uniformly expanding spherical wavefront.

∴ Eqn. of wavefront as seen by observer at O in ref. frame S

$$\Rightarrow \quad x^2 + y^2 + z^2 = c^2t^2 \quad \text{—— (1)}$$

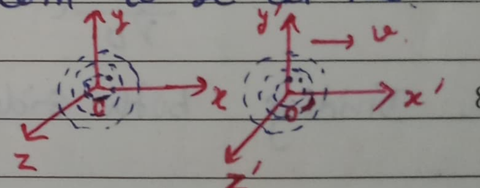
& Eqn of wavefront as seen by observer at O' in ref. frame S' is

$$x'^2 + y'^2 + z'^2 = c'^2 t'^2 = c^2 t'^2 \quad \text{—— (2)}$$

{∵ c = vel. of light is same in both f.o.r.

In order that both the observers may seem to be at the centre of same wavefront.

∴ Eqn (1) = Eqn (2)



Now by Galilean Transformation,

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$\therefore \quad x'^2 + y'^2 + z'^2 = c^2 t'^2$$

$$\Rightarrow (x - vt)^2 + y^2 + z^2 = c^2t^2$$

$$\Rightarrow x^2 - 2xvt + v^2t^2 + y^2 + z^2 = c^2t^2 \neq \text{Eqn. (1)}$$

⇒ [Galilean Transformation fails if Constancy of value of c be assumed

So we require a new Transformation Eqn. known as Lorentz Transformation Eqn.

$$\boxed{x' = \gamma(x - vt) \ , \quad y' = y \ , \quad z' = z.}$$

$\because$ By Inverse Transformat. $\quad x = \gamma(x' + vt')$

$$\Rightarrow x = \gamma[\gamma(x - vt) + vt']$$

$$\Rightarrow x = \gamma[\gamma x - \gamma vt + vt']$$

$$\Rightarrow x = \gamma^2 x - \gamma^2 vt + \gamma vt'$$

$$\Rightarrow \gamma vt' = x - \gamma^2 x + \gamma^2 vt$$

$$\Rightarrow \gamma vt' = x(1 - \gamma^2) + \gamma^2 vt$$

$$\Rightarrow t' = \frac{x}{\gamma v}(1 - \gamma^2) + \gamma t$$

Now $\quad x' = ct'$

$$\gamma(x - vt) = c\left[\frac{x}{\gamma v}(1 - \gamma^2) + \gamma t\right]$$

$$\Rightarrow \gamma x - \gamma vt = \frac{cx}{\gamma v}(1 - \gamma^2) + c\gamma t$$

$$\Rightarrow \gamma x - \frac{cx}{\gamma v}(1 - \gamma^2) = c\gamma t + \gamma vt$$

Dividing both sides by γ —

$$x - \frac{cx}{\gamma^2 v}(1 - \gamma^2) = ct + vt$$

$$x\left[1 - \frac{c}{v}\left(\frac{1}{\gamma^2} - 1\right)\right] = ct\left[1 + \frac{v}{c}\right]$$

$$\Rightarrow 1 - \frac{c}{v}\left(\frac{1}{\gamma^2} - 1\right) = 1 + \frac{v}{c} \qquad \left\{\because x = ct\right.$$

$$\Rightarrow 1 - 1 - \frac{v}{c} = \frac{c}{v}\left(\frac{1}{\gamma^2} - 1\right)$$

$$\Rightarrow -\frac{v^2}{c^2} = \left(\frac{1}{\gamma^2}-1\right)$$

$$\Rightarrow \frac{1}{\gamma^2} = 1-\frac{v^2}{c^2}$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1-v^2/c^2}}}$$

$$\therefore t' = \frac{x}{\gamma v}(1-\gamma^2) + \gamma t$$

$$t' = \frac{x}{\gamma v}\left[1-\frac{1}{1-v^2/c^2}\right] + \gamma t$$

$$t' = \frac{x}{\gamma v}\left[\frac{-v^2/c^2}{1-v^2/c^2}\right] + \gamma t = \gamma\left[\frac{x}{\gamma^2 v}\left(\frac{-v^2/c^2}{1-v^2/c^2}\right) + t\right]$$

$$t' = \gamma\left[t - \frac{xv}{c^2}\right]$$

Lorentz Transf. Eqn.

$$\therefore \begin{cases} x' = \gamma(x-vt) \\ y' = y \\ z' = z \\ t' = \gamma\left[t-\frac{xv}{c^2}\right] \end{cases}$$

Relativistic Transformation Eqn.

Let $v \lll c$

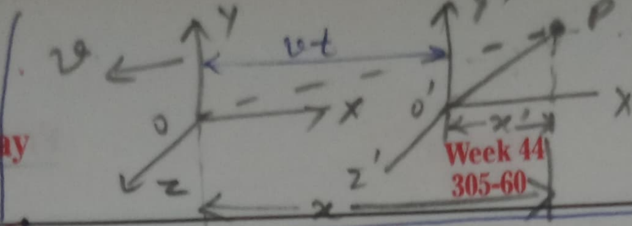
$$\Rightarrow \frac{v}{c}\to 0 \Rightarrow \gamma \to 1$$

Galilean Transf.

$$\therefore \begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

Hence we get that Galilean or Newtonian Physics is just a particular case of Relativistic Physics. Since Vel v for earth being very small than c, $\therefore$ Galilean or Newtonian Physics is thus perfectly valid for almost all our practical purposes. The Relativistic Effects come into picture only at very high values of v, ie when they are comparable with c. Eg: for e^-, protons & neutrons.

NOTE



$x = \gamma(x' + vt')$

Inverse Transformation Eqn.

Lorentz Trans.

$x = \gamma(x' + vt')$
$y = y'$
$z = z'$
$t = \gamma\left[t' + \dfrac{vx'}{c^2}\right]$

Galilean Transf.

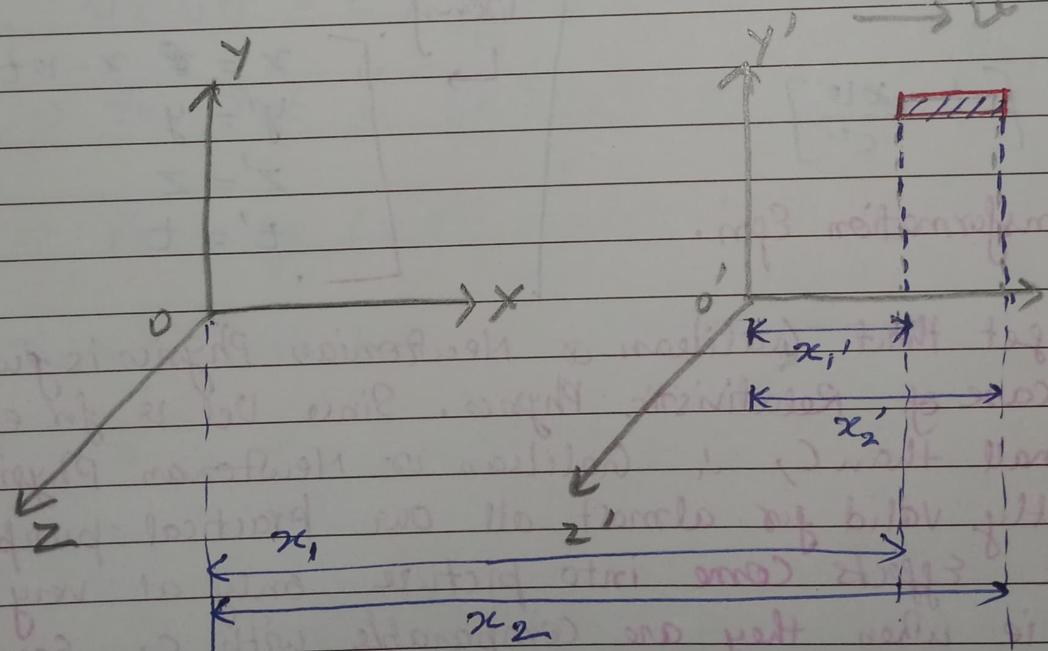
$x = x' + vt'$
$y = y'$
$z = z'$
$t = t'$

⇒ Measurement of space and time are by no means absolute but are dependent upon the relative motion b/w observer and phenomenon observed.

Ex. of Relativistic Effects

1 Length Contraction —

proper length — length of rod as measured in f.o.r. at rest w.r.t. observer.



x_1 & x_2 → Coordinates of two ends when rod is at rest w.r.t observer

∴ ~~Relative~~ Length of Rod $= L_0 = x_2 - x_1$

Let S' is moving with v w.r.t S & x_1' & x_2' be the coordinates of rod w.r.t S' at time t'.

$\therefore$ Length of Rod $= L_0 = x_2' - x_1'$ = Proper length of Rod.

Now $L_0 = x_2' - x_1'$

$$= \frac{x_2 \cancel{+ vt'}}{\sqrt{1 - v^2/c^2}} - \left[\frac{x_1 \cancel{+ vt'}}{\sqrt{1-v^2/c^2}}\right]$$

$$= \frac{x_2 - x_1}{\sqrt{1-v^2/c^2}} \qquad \left\{ \because x_2' = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}}, \quad \because x_1' = \frac{x_1 - vt}{\sqrt{1-v^2/c^2}} \right.$$

$$L_0 = \frac{L}{\sqrt{1-v^2/c^2}}$$

$\Rightarrow$ $\boxed{L_0 = \gamma L}$

$\Rightarrow$ $\boxed{L = L_0 \sqrt{1-v^2/c^2}}$ ↑ proper length.

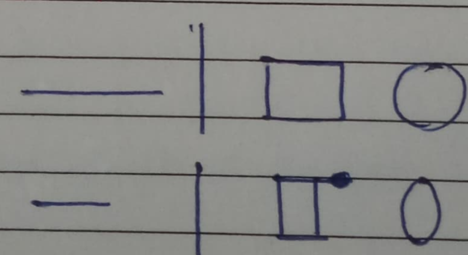
$\Rightarrow$ higher the value of v i.e. faster the motion of the rod (or the reference frame carrying it) w.r.t the stationary observer, the shorter its length compared with its proper length.

[if $v = c$ $\Rightarrow$ $L = 0$ $\Rightarrow$ If the rod is moving with the vel. of light, its length would be zero.

This shortening or contraction in the length of an object along its dirn of motion is known as Lorentz-Fitzgerald contraction.

Ref. frame at rest

Ref. frame in motion ($\rightarrow$)



Time Dilation

Suppose we have 2 f.o.r. S & S' & S' is moving with uniform vel v relative to S, along the $+x$ direction.
Suppose that two identical clocks are carried by the two reference frames at their origin O & O' such that they show zero time when just cross each other.

Let t_1' and t_2' be the time period of two events in frame S'
& t_1 & t_2 be the time period of ~~frame~~ " " " " S.

$$\therefore \Delta t' = t_2' - t_1' \quad \& \quad \Delta t = t_2 - t_1 \leftarrow \text{Proper } \cancel{\text{Relative}} \text{ time.}$$

↑ Proper time
Relativistic

Now
$$\Delta t' = t_2' - t_1'$$
$$= \gamma\left(t_2 - \frac{vx}{c^2}\right) - \gamma\left(t_1 - \frac{vx}{c^2}\right)$$
$$\Delta t' = \gamma(t_2 - t_1)$$
$$\Delta t' = \gamma \Delta t$$

$$\Rightarrow \Delta t' > \Delta t \quad \because \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} > 1.$$

⇒ moving clock runs more ~~slowly~~ faster than a stationary clock.

⇒ The time interval $\Delta t'$ b/w two events occuring at a given point in the moving frame S' appears to be longer or dilated by a factor γ to the observer in the stationary frame S.